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Crack Driving Force in Geometrically Nonlinear Orthotropic Functionally Graded Materials

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ABSTRACT

The objective of this contribution is the exploitation of the notion of material forces in the computational fracture mechanics of functionally graded materials. To this end we consider the framework of orthotropic geometrically linear and non–linear hyperelasticity and investigate the spatial and material settings that lead to either spatial, i.e. physical, or material forces, respectively. Contrary to physical forces, material forces act on the material manifold, thus essentially representing the tendency of defects, like e.g. cracks, to move relative to the ambient material.

In the case of hyperelastic functionally graded materials the corresponding stored energy density function W_0 explicitly depends on the placement $\boldsymbol{X}$ of a 'physical particle' in the material configuration $\mathcal{B}_0$. Thus the appearance of distributed material volume forces are induced by the explicit material gradient of the stored energy density function. These additional material volume forces are closely related to the commonly used interaction integral, see e.g. [1,2] and vanishes identically in the case of homogeneous materials. Whereas these interaction integrals are resticted to linear elastic problems the proposed method is also applicable in the case of large strain hyperelasticity.

Using a Galerkin discretization of the corresponding balance of momentum equation in material space we obtain a consistent and algorithmically straight-forward methodology for the determination of discrete material forces which are directly related to relevant quantities in fracture mechanics of functionally graded materials, like the crack driving force, i.e. the vectorial J-integral.

We characterize the orthotropic material in the material configuration, i.e. the undeformed configuration, with two orthogonal vectors $\boldsymbol{A}_1$ and $\boldsymbol{A}_2$, the so called fiber directions. These to vectors define the structural tensors $\boldsymbol{M}_1 = \boldsymbol{A}_1 \otimes \boldsymbol{A}_1$ and $\boldsymbol{M}_2 = \boldsymbol{A}_2 \otimes \boldsymbol{A}_2$. Thus the Helmholtz or stored energy density depends in the case of orthotropy on the right Cauchy–Green strain tensor $\boldsymbol{C} = \boldsymbol{F}^t \cdot \boldsymbol{F}$, with $\boldsymbol{F}$ the deformation gradient, and the two structural tensors $\boldsymbol{M}_1$ and $\boldsymbol{M}_2$. We reformulate this Helmholtz energy in terms of the seven invariants of $\boldsymbol{C}, \boldsymbol{M}_1, \boldsymbol{M}_2$ by the representation theorem of isotropic tensor functions. The seven invariants are: $I_1 = \operatorname{tr} \boldsymbol{C}, I_2 = \operatorname{tr} \operatorname{cof} \boldsymbol{C}, I_3 = \det \boldsymbol{C}, I_4 = \boldsymbol{C} :$

$\boldsymbol{M}_1, I_5 = \boldsymbol{C}^2 : \boldsymbol{M}_1, I_6 = \boldsymbol{C} : \boldsymbol{M}_2$ and $I_7 = \boldsymbol{C}^2 : \boldsymbol{M}_2$. So that we have $\psi_0(\boldsymbol{C}, \boldsymbol{M}_1, \boldsymbol{M}_2) = \psi_0(I_1, I_2, I_3, I_4, I_5, I_6, I_7)$. By the introduction of an additional explicit dependency of the Helmholtz energy on the material placement $\boldsymbol{X}$, we are able to describe an orthotropic functionally graded material by $\psi_0 = \psi_0(I_1, I_2, I_3, I_4, I_5, I_6, I_7; \boldsymbol{X})$. This explicit dependency induces the so called material volume forces $\boldsymbol{B}_0 = \partial_{\boldsymbol{X}} \psi_0$, which are responsible for the loose of path independence of the J-integral. With the help of the Material Force Method, see [3], we are able to calculate these material volume forces and therefore we can calculate the crack driving force in a geometrically nonlinear orthotropic hyperelastic functionally graded material with high precision. A small example is given in the figure below.

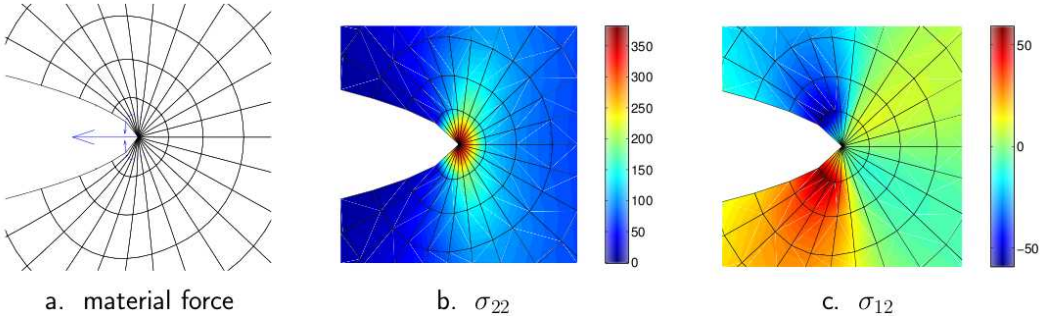

Figure 1: Material forces, i.e. crack driving forces and stresses in the vicinity of a mode I crack at large strains.

Numerical results for a number of model problems in fracture mechanics of geometrically linear and non–linear orthotropic functionally graded materials are presented and compared to results obtained from the literature.

REFERENCES

[1] B. N. Rao and S. Rahman. An interaction integral method for analysis of cracks in orthotropic functionally graded materials. *Computational Mechanics*, 32:40–51, 2003.

[2] J.-H. Kim and G. H. Paulino. The interaction integral for fracture of orthotropic functionally graded materials: evaluation of stress intensity factors. *International Journal of Solids and Structures*, 40:3967–4001, 2003.

[2] R. Denzer, F. J. Barth, and P. Steinmann. Studies in elastic fracture mechanics based on the material force method. *International Journal for Numerical Methods in Engineering*, 58:1817–1835, 2003.