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SHAPE FINDING OF TAUT STRUCTURES BY THE NATURAL FORCE DENSITY METHOD

***Ruy M.O. Pauletti & Paulo M. Pimenta**

Polytechnic School, University of São Paulo
P.O. Box 61548, 05424-970 São Paulo, Brazil
{pauletti, ppimenta}@usp.br

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ABSTRACT

Since Frei Otto's pioneering works in the 1950's, taut structures constitute an important research field in architecture and engineering. They are light, elegant and effective structures, whose applications range from large stadium roofs and high-rise building walls to pneumatic furniture or aerospace equipment.

Taut structures are characterized by profusion of possible equilibrated initial configurations, and, for this reason, it is difficult to define their geometric shape a priori. The design of a taut structure involves the determination of an initial equilibrated configuration or **viable configuration**, which encompasses the structure's shape and the associated stress field. The viable configuration must accommodate both architectonic (form and function) and structural requirements (strength and stability). The design of taut structures is consequently integrated to their analysis, in a process that includes procedures for shape finding, patterning and load analysis. Some references on the design of taut structures are [1],[2],[3],[4] and [5].

A versatile way to pose the overall design process of taut structures is via the Finite Element Method (FEM). It directly provides, besides a viable shape, also a map of the stresses to which the structure is subjected. It is also adequate to determine the behavior of the structure under design loads, as well as to transfer data to the patterning routines. On the other hand, procedures based on the FEM or in other forms of structural analysis result in nonlinear analyses, and require specification of a convenient initial geometry, load steps and boundary conditions, which are not always defined *ab initio*.

An alternative method for finding viable configurations, which avoids the problems related to nonlinear analysis, is given by the force density method, which was first proposed in the context of cable nets ([6],[7],[8]). Some analogous procedures for the shape finding of membrane structures have already been proposed in the literature; see e.g. [9] and [10]. However, these procedures are not linear and require iterations for solution. In this way, the advantage of the original force density method is lost, and there is no clear reason to replace a nonlinear structural analysis by another nonlinear procedure.

With the aid of the natural approach introduced by Argyris ([11]) for the Finite Element Method, this work presents an extension of the force density method for the initial shape

finding of cable and membrane structures, which leads to the solution of a system of linear equations. This method is called the **natural force density method** (see [12]), and preserves the linearity of the original force density method. Furthermore, if it is applied iteratively in the lines prescribed herein, it may lead to a viable configuration with a uniform, isotropic plane Cauchy stress state. This means that a minimal surface for a membrane can be achieved through a succession of viable configurations. This is an advantage, if compared to Newton's Method, which may also converge to a minimal solution, but through a series of unfeasible configurations. In case of membranes, the Newton's Method has the additional drawback of requiring the solution of unsymmetrical linear systems. Several numerical examples demonstrate the simplicity and robustness of the method.

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