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SUPERPOSITION OF SOFTENING LAWS TO ACCOUNT FOR MULTIPLE TOUGHENING MECHANISMS

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ABSTRACT

To predict the propagation of damage in quasi-brittle materials such as composites, it is necessary to define damage evolution laws that account for the fracture energy dissipated in each damage mode[1, 2]. A thermodynamically consistent softening response can be obtained using bilinear or exponential traction displacement laws defined by three variables: an initial penalty stiffness K_p, a maximum strength σ_c, and a toughness G_c. However, many common fracture processes exhibit a toughness that increases with crack length – a phenomenon denoted as the R-curve effect.

The objective of the present work is to examine the relations between a material's R-curve, its fracture process zone length, the shape of the traction/displacement softening law, and the initiation and propagation of fracture. A procedure that accounts for toughening effects by superposing bilinear cohesive elements is proposed (Fig. 1). To determine the strength ratio $n = \sigma_{c1} / \sigma_c$ and the toughness ratio $m = G_1 / G_c$ that define the proportions of two superposed bilinear softening laws, the following expressions were developed for the R-curve and its associated process zone length:

$$G_R(\Delta a) = \underbrace{n\left[\frac{1}{\gamma}\frac{\sigma_c^2}{E}\Delta a\right]}_{\leq G_1} + \underbrace{(1-n)\left[\frac{1}{\gamma}\frac{\sigma_c^2}{E}\Delta a\right]}_{\leq G_2}, \quad \text{and} \quad l_p = \gamma\ \frac{1-m}{1-n}\frac{EG_c}{\sigma_c^2}$$

where E is the Young's modulus of the laminate and γ is a constant. A numerical investigation confirms that the R-curves predicted by this equation with a constant $\gamma = 0.6$ correlate well with the R-curves extracted from finite element analyses with superposed cohesive elements for all the material properties considered.

These proposed equations were used to determine the coefficients n and m that are necessary to represent the R-curve measured by Pinho[3] using the Compact Tension specimen shown in Fig. 2. The experimental energy release rate (ERR) results shown in Figure 3 were obtained using the Modified Compliance Calibration (MCC) technique using the experimental load-deflection values and a compliance curve calculated by finite elements. The results indicate that the ERR for fracture initiation is about half of the ERR for propagation and that it takes about 11 mm of crack growth to reach the steady state propagation of G_c=180 kJ/m^2.

Two models of the CT specimen were created: one denoted "FEM 1" with a single row of bilinear cohesive elements and the second denoted “FEM 2” with a row of two superposed elements. The coefficients n=0.94 and m=0.56 were determined by using the proposed equation for G_R and l_p=11mm to approximate the upper bound of the R-curve of Pinho’s experiments. The R-curves for the two finite element analyses obtained with the MCC technique are shown in Fig. 3. The R-curve for the model “FEM 2” correlates well with the analytical curve G_R and with the experimental results.

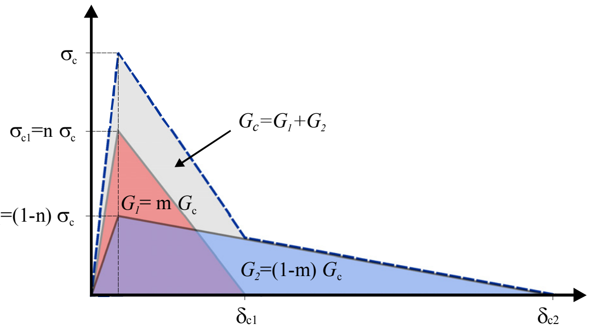

Fig. 1. Superposition of two bilinear softening laws to represent R-curve toughening.

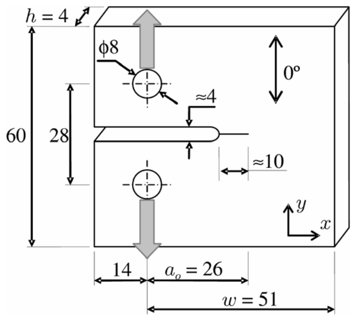

Fig. 2. CT specimen for measuring toughness of fiber fracture. (after Pinho[3])

By comparing the load-displacement responses in Fig. 4 for models “FEM 1” and “FEM 2”, it can be observed that the use of superposed elements reduces the error in the predicted strength of the CT specimen from 29% to 2.8%. This improvement in accuracy is obtained without modifying any material property used in the analysis and without any additional modeling effort beyond that required to find the strength ratio n and the toughness ratio m. Finally, the proposed approach relies on available bilinear cohesive elements so additional finite element development is unnecessary.

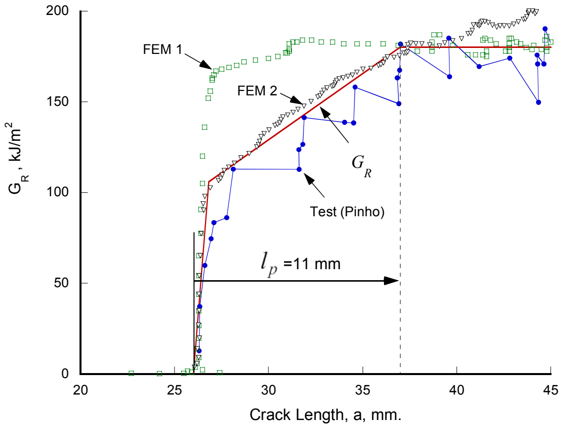

Fig. 3. R-curves for analytical model, finite element models, and experiment.

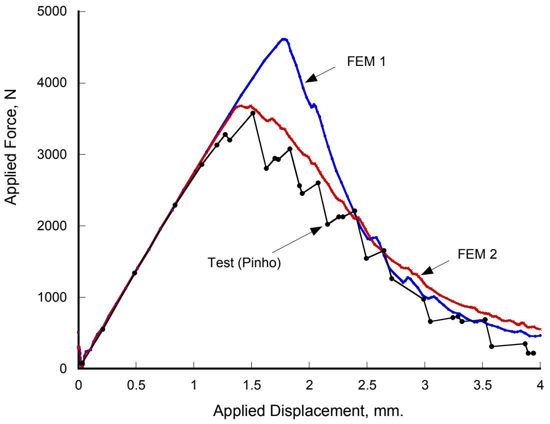

Fig. 4. Experimental and finite element load-displacement response of CT specimen.

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