8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineeering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

Parallel Solution of the Generalized Dirichlet-Neumann Map for Elliptic PDEs on Regular Polygon Domains

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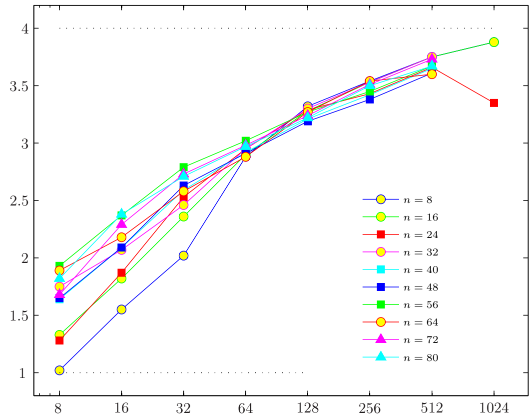
Key Words: *Elliptic PDEs, Collocation, Block Circulant Matrices, Fast Fourier Transform (FFT), Shared-Distributed Memory Parallel Machines, Grid/Cluster Computing.*

ABSTRACT

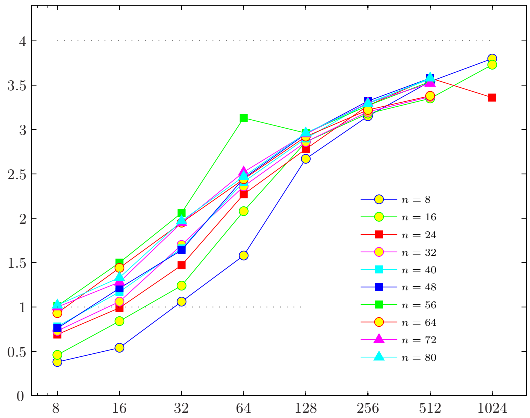
Recently, in [2,3], a new and novel unified approach was introduced for analyzing linear and integrable nonlinear PDEs in two dimensions. Central issue to this approach is a generalized Dirichlet-Neumann map, characterized through the solution of the so-called *global relation*, namely an equation, valid for all values of a complex parameter k, coupling specified known and unknown values of the solution and its derivatives on the boundary.

For a large class of boundary value problems, the global relation can be solved analytically, and hence the generalized Dirichlet-Neumann map can be constructed in closed form. However, for general boundary value problems, the global relation must be solved numerically. For this, in [4], a well conditioned and fast convergent collocation-type numerical method was developed and studied for the numerical solution of the Generalized Dirichlet-Neumann map associated to the generic model problem of Laplace's equation on an arbitrary convex polygon domain. For the case of regular polygon domains, with the same type of boundary conditions on all sides, we have (cf. [5]) rigorously studied the properties of the associated collocation coefficient matrix revealing its *Block Circulant* structure. And as the block circulant property is strongly connected with the Discrete Fourier Transforms (cf. [1]), the produced linear system can be solved efficiently using FFTs (cf. [6]). The development of a parallel algorithm for this computational task is the main problem we are addressing in the work herein. The parallel algorithm we present is realized, through MPI programming, on two parallel systems: (a) on a shared-distributed memory computer with 8 processors and (b) on a cluster of 4 nodes with 2 processors each. The cluster uses a local ethernet interconnection for its nodes of either 100Mbps or 1 Gbps.

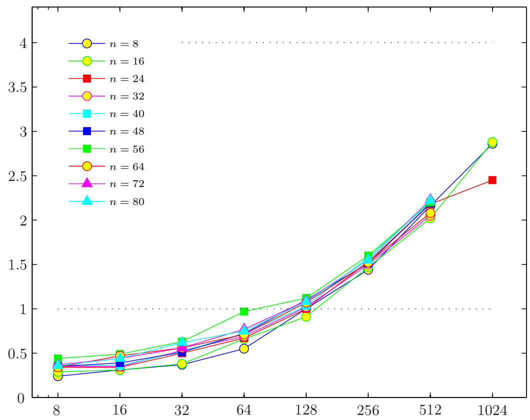
Our implementation is further studied through extensive numerical experimentation accompanied with computation/communication and speedup measurements (see for example Figure 1 below). Through this study we are able to draw conclusions for the performance of our implementation as well as to evaluate and compare the different parallel architectures used.



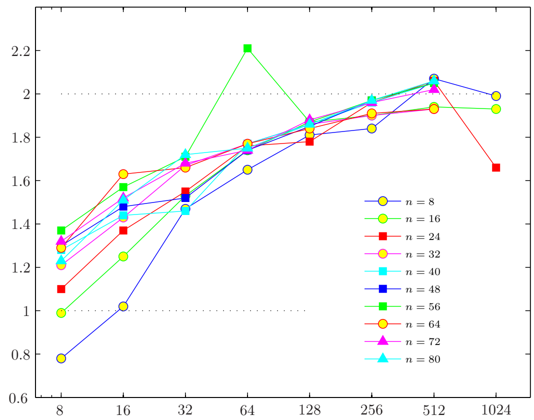
(a) Shared Memory, 4 processors.



(b) Gbps Cluster, 4 processors.



(c) Mbps Cluster, 4 processors.



(d) Cluster, 2 processors.

Figure 1: The speedup for the solution of the Dirichlet-Neumann map on a variety of regular polygons (number of sides $n = 8, 16, 24, 32, 40, 48, 56, 64, 72, 80$) with discretization size (the number of basis functions per side) $8, 16, 32, 64, 128, 256, 512, 1024$.

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