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Numerically repeated support splitting and connecting phenomena in the flow through an absorbing medium.

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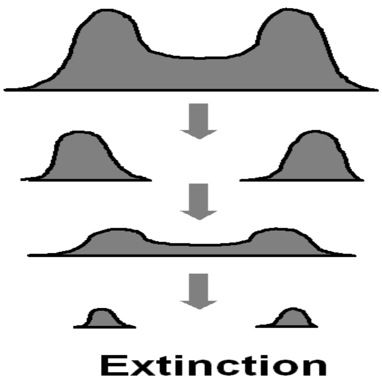
ABSTRACT

Numerical experiments to nonlinear heat equations suggest several interesting phenomena. One of them is the occurrence of *numerical support splitting phenomena* caused by the interaction between diffusion and absorption. Here the support means the region where $v > 0$, and the *interface curves* appear between the hot state ($v > 0$) and the cold one ($v = 0$). The model equation which describes such a process is written in the form of the following initial value problem:

$$v_t = \Delta v^m - cv^p, \qquad x \in \mathbf{R}^2, \quad t > 0, \tag{1}$$
$$v(0,x) = v^0(x), \qquad x \in \mathbf{R}^2, \tag{2}$$

where $m > 1$, $0 < p < 1$, c is a positive constant, v denotes the temperature, $-cv^p$ describes volumetric absorption, and $v^0(x) \in C^0(\mathbf{R}^2)$ is nonnegative and has compact support.



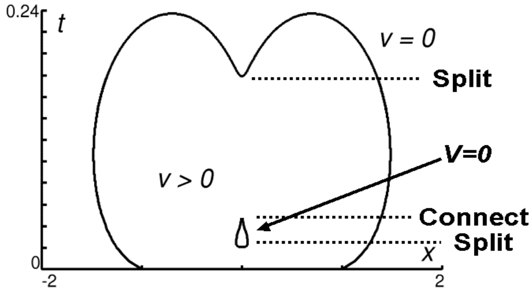


Figure 1: Support re-splitting phenomena. The left figure shows the illustration. The right one shows numerical interfaces for $v_t = (v^{1.5})_{xx} - 5v^{0.5}$.

In the 1-dimensional case, Rosenau and Kamin([7]) showed the possibility of the support to split into several disjoint sets. But the theoretical justification is not discussed. Chen, Matano and Mimura ([1])

constructed the solution $v(t,x)$ such that the support of $v(t,\cdot)$, while initially connected, split in a finite time. From numerical points of view, by using the L_1, L_∞ estimates derived from the properties of the finite difference schemes, Nakaki and Tomoeda ([6]) obtained the sufficient condition under which the support begins to split in the specific case $m+p=2$.

According our numerical method which tracks the interface curves, the most striking property is the occurrence of *numerical support splitting and connecting phenomena*, which are repeated and finally become extinct ([8],[9]). It is difficult to justify whether *such phenomena* are true or not, because the space mesh and the time step are sufficiently small but not zero. This motivates us to investigate such phenomena from both numerical and analytical points of view.

In this talk we show our numerical method and experiments, and try to give the initial function in $\mathbf{R}^n$ $(n=1,2)$ for which such phenomena occur at least m times for an arbitrary fixed integer $m(\geq 2)$ in the case $m+p=2$. For this end it is needed to extend the L_1, L_∞ estimates to the 2-dimensional case. Unfortunately, we are unable to obtain this extension at this present. To avoid such difficulties we use the properties of the particular solutions ([2],[3],[4],[5]) and the comparison theorem.

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