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CUR model reduction of mechanical structures

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ABSTRACT

Recently, CUR decompositions [1] has appeared as a novel tool in the numerical linear algebra scenario. It is a technique where Fast Monte-Carlo algorithms are used to obtain a compressed approximation of the original matrix A. Briefly, assigned a probability p_i and q_j to each row and to each column of the matrix A, the algorithm chooses c columns and r rows, which become the matrices C and R, respectively. Then, it computes the $c \times r$ matrix U from approximate SVD computations [2] and approximate matrix multiplications [3]. Indeed, if the probabilities $\{p_i\}$ and $\{q_j\}$ are chosen correctly, the matrix $A' = CUR$ is a good k-rank approximation of A, with $1 \leq k \leq \min\{r, c\}$.

Here we consider to apply CUR decompositions in the model reduction of large-scale finite element models, which have important applications to mechanical and civil structures. For a comprehensive survey of model reduction see e.g. [4].

In a finite element model, CUR decompositions can give a reduced order model in nodal, i.e. physical, cohordinates. This is recognized as an uncompletely explored issue, and merits some attention. Moreover, from an application perspective, it is important to note that the independent choice of the c column indices and of the r row indices means that the measurement nodes and the forcing nodes in the reduced model are independent subsets of the complete set of nodes in the original model.

It is well known that even in simple structures, like a cantilevered beam, some nodes may have null displacements for some dominant vibrating modes and their position changes with the physical settings. This is even more true for more complicated structures. CUR decompositions can be a valid tool for determining with a Monte-Carlo algorithm the best nodal candidates for the reduction.

Moreover, they combine a selection of nodal variables (columns) with a selection of weighting functions (rows). This fact takes a special meaning in a finite element model, e.g. since the shape- and the weighting functions have local support. This allows us to go beyond the general method and to present what really means a CUR decomposition in a finite element context. In particular, we try to show how the known and deterministic structure of the discrete finite element model can be used in support of the Monte-Carlo technique.

We present some results on a one- and two-dimensional scalar field problem, discretized by linear finite elements. It is clear that the computational advantage arise when the problem is large-scale, but here

we want to show its effectiveness in a finite element model reduction. Therefore, the test problem here considered is small scale, and we compare the responses of the original model, a CUR-reduced model and an SVD-reduced model of the same size.

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