

## FORCE-BASED VERSUS DISPLACEMENT-BASED FORMULATIONS IN THE CYCLIC NONLINEAR ANALYSIS OF REINFORCED CONCRETE FRAMES

\*António A. Correia<sup>1</sup>, João P. Almeida<sup>2</sup> and Rui Pinho<sup>3</sup>

<sup>1</sup> ROSE School  
aaraujo@roseschool.it

<sup>2</sup> ROSE School  
jpacheco@roseschool.it

<sup>3</sup> University of Pavia  
rui.pinho@unipv.it

### ABSTRACT

Historically, the vast majority of finite elements used for the nonlinear analysis of framed structures are based on displacement formulations (also known as stiffness formulations). These finite elements rely on an approximation for the displacement field throughout the frame element length, from which strains, stresses and stress resultants are computed. The easiness of implementation and the need of an iterative process only at the structural level are the main advantages for using these stiffness-based formulations in the nonlinear analysis of structures. Given the approximate nature of the displacement interpolation functions, the displacement field on the element is exact only if the frame element is prismatic, with linear elastic behaviour and the loading consists only of nodal loads. If any of these premises is not satisfied, the results obtained by this formulation will be stiffer than the real ones. Hence, the advantages of displacement-based formulations tend to disappear when inelastic behaviour is modelled. This issue is of special relevance in the seismic analysis of structures.

Inversely, force or flexibility-based formulations use an approximation for the stress resultants field throughout the frame element length, which strictly satisfies equilibrium conditions and is exact independently of the nonlinearity in the material behaviour. Hence, the difficulties arising in displacement-based formulations are inexistent in this framework. The main disadvantage of this approach is the need of a three-level iterative procedure: structure, element and cross-section. However, recent work has shown that this iterative procedure can be transformed in a two level or even a single level iterative procedure, without loss of accuracy. But even this issue of a smaller computational cost of the displacement-based formulations is opposed by the need of adopting more elements per frame element in order to obtain similar results to the force-based formulations. Another advantage of the latter is the easiness of considering span loading.

To validate and evaluate both formulations, numerical and experimental results of cyclic tests on bridge piers are compared, including strain localisation phenomena. The superiority of force-based formulations in comparable conditions is established.