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High order P3 Hermite triangular finite element for transport and incompressible flow problems

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ABSTRACT

The P3-Hermite triangular finite element, despite its advantages with respect to the P3-Lagrange element in terms of number of degrees of freedom for a given mesh, has practically fallen into oblivion. The purpose of the present research was to evaluate its suitability and performance for steady transport and incompressible flow problems.

First, the element fourth-order accuracy was verified for Laplace's equation on domain with straight and curved boundaries.

Then, the advection-diffusion equation was considered. It is well known that Galerkin finite element discretizations of advection-diffusion problems suffer from instabilities in the advection dominated limit, and thus need to be stabilized. Stabilization was achieved using a classical SUPG approach, but it was noticed that smaller values of the stabilization coefficient were required.

In a third step, Stokes' flow was considered. Galerkin finite element discretizations of this problem with equal order velocity/pressure interpolations are also know to suffer from instabilities, namely spurious pressure modes. Although it was verified numerically that the Galerkin P3-H/P3-H discretization does not satisfy the stability condition (LBB condition), spurious pressure instabilities were hardly ever observed in actual computations of the lid-driven cavity problem.

Finally, the P3-H finite element was applied to the incompressible Navier-Stokes equations, for the (regularized) lid-driven cavity test case. Excellent results were obtained, as illustrated in Fig. 1, which compares the present results with the reference numerical solution of Ghia and al. [1]. Very good agreement is observed, despite the (very) small number of degrees of freedom (218 nodes, 362 elements $\Rightarrow$ $3 \times 218 + 362 = 1016$ degrees of freedom).

In conclusion, the results obtained are very encouraging. Despite a somewhat higher implementation complexity, the P3-Hermite finite element appears to be highly suitable for incompressible flow simulations.

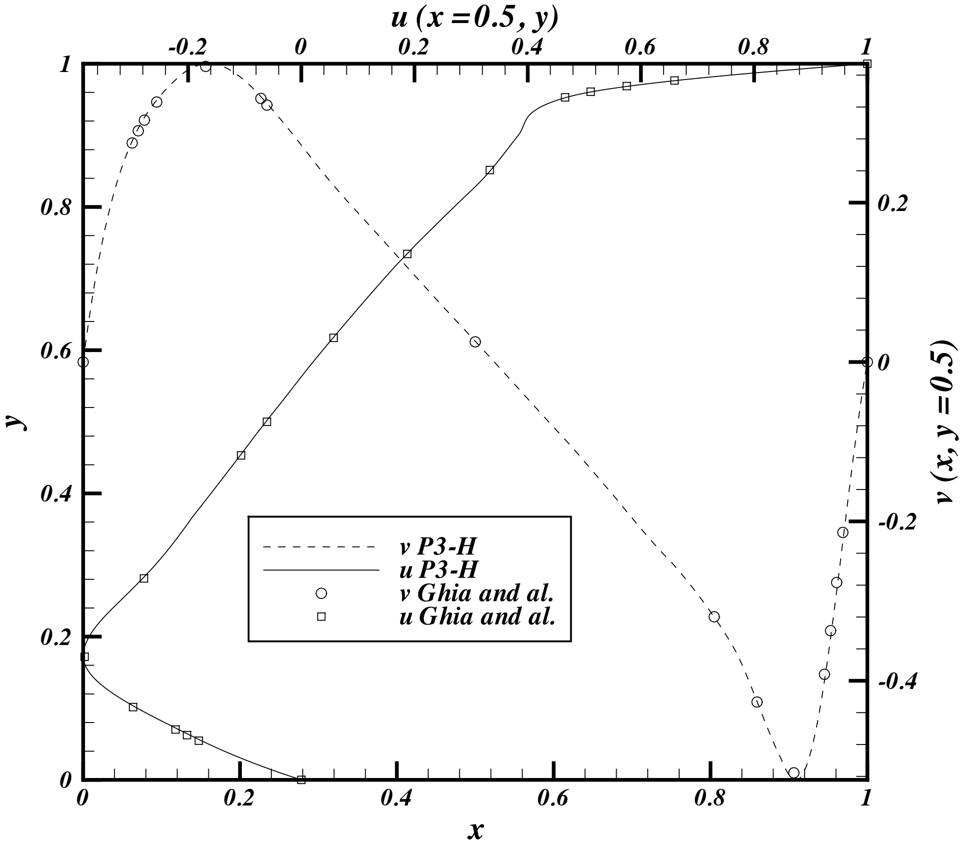

Figure 1: Lid-driven cavity ($Re = 1000$): comparison between present results and Ghia and al. [1]

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