8th. World Congress on Computational Mechanics (WCCM8)
5th. European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 – July 5, 2008
Venice, Italy

NUMERICAL IMPLEMENTATION OF THE FORCE METHOD IN THE ELASTOPLASTIC ANALYSIS OF FRAMES

***Konstantinos V. Spiliopoulos and Theodoros N. Patsios**

Institute of Structural Analysis & Aseismic Research
National Technical University of Athens
Zografou 157-80, Athens, Greece
kvspilio@central.ntua.gr.

Key Words: *Mathematical programming, Numerical Methods, Elastoplastic Analysis, Plastic Hinges, Planar Frames.*

ABSTRACT

The response of a linearly-elastic perfectly-plastic frame subjected to proportional loading, may be obtained with the use of incremental loading steps. If we take an increment of the applied loading $\Delta\mathbf{f} = (\Delta\gamma)\mathbf{f}'$ from an initial state, where γ is the proportional parameter, we may determine an increment of the moments that develop using the force (or mesh) description of statics:

$$\Delta\mathbf{m} = \mathbf{B}\mathbf{p} + (\Delta\gamma)\mathbf{B}_o\mathbf{f}' \qquad (1)$$

where the first term is due to the indeterminacy of the structure and $\mathbf{p}$ is a statical basis.

These two matrices are established, in an automatic way, using concepts from graph theory like a minimum path technique between two nodes of a graph (Spiliopoulos [1]).

Assuming that plasticity is concentrated at the two ends of each member of the frame, the total rotations may be decomposed into an elastic and a plastic part. A rigid plastic nonholonomic behaviour may be assumed for the plastic part. Such a behaviour (pictured in Fig.1 for a positive plastic rotation) may be expressed through the use of the plastic potential $\mathbf{y}_*$:

$$\mathbf{y}_* \geq \mathbf{0} \quad , \quad \mathbf{y}_*^T \Delta\boldsymbol{\theta}_* = 0 \quad \Delta\boldsymbol{\theta}_* \geq \mathbf{0} \qquad (2)$$

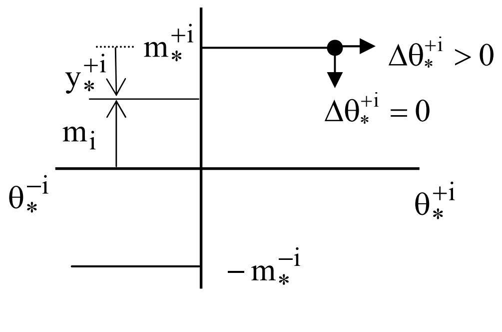

Fig.1: Rigid-plastic behaviour

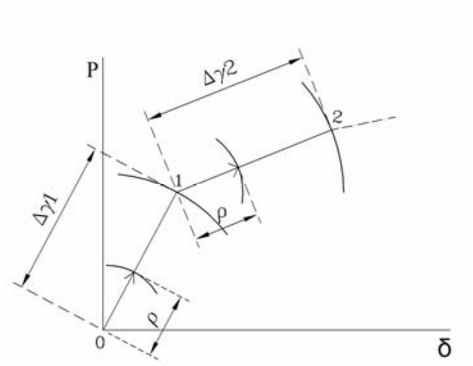

Fig.2: Numerical strategy

Equations (1), (2) together with the compatibility and the static admissibility conditions, expressed also through the plastic potential, may be seen as Kuhn-Tucker constraints leading to the following quadratic programming problem (QP):

$$\text{Min} z = \frac{1}{2}\mathbf{p}^{T}(\mathbf{B}^{T}\mathbf{F}\mathbf{B})\mathbf{p} + \Delta\gamma(\mathbf{B}^{T}\mathbf{F}\mathbf{B}_{o}\mathbf{f}')\mathbf{p} \tag{3}$$

subject to:

$$(\mathbf{N}^{T}\mathbf{B})\mathbf{p} \le (\mathbf{m}_{*} - \mathbf{N}^{T}\mathbf{m}_{pr}) - \Delta\gamma(\mathbf{N}^{T}\mathbf{B}_{o}\mathbf{f}') \tag{3a}$$

Expression (3a) is the condition of static admissibility at the current load increment, with the term $\mathbf{m}_{pr}$ being the total moments obtained from the previous increments. $\mathbf{m}_{*}$ denote the plastic moment capacities of the cross sections both in tension and compression, **F** collects the flexibility matrices of the unassembled members and **N** is an incidence matrix.

The above QP program is a parametric one, since, to find the unknowns **p**, the papameter $\Delta\gamma$ should also be supplied. This papameter may be estimated requiring that each load increment ends with the formation of a new plastic hinge. Maier [2] was the first to present the above program in the form of an equivalent parametric linear complementarity problem (PLCP). Smith [3] has suggested the solution of this problem using the Wolfe-Markowitz algorithm. The PLCP problem contains both static and kinematic variables and its solution requires operations on both of these sets.
On the contrary, the QP program contains only static variables and a solution of this program would be preferable. In the present work a novel numerical strategy to solve directly the QP program is suggested.

The main idea of this strategy is to disassossiate the parameter $\Delta\gamma$ from the solution of the QP. This may be accomplished if we find a direction on which the sought solution lies. This direction may be determined by replacing $\Delta\gamma$ with a given relatively small number ρ and then solving this QP program using any existing algorithm (e.g. [4]). In this way, a set of statically admissible bending moments and a set of elastic/plastic rotations may be established. The real current $\Delta\gamma$ could then be evaluated as the smallest value these bending moments should be multiplied with, so that a new plastic hinge forms. Then, in order to get the true solution of the current step, we merely multiply the above mentioned sets by this factor since a step between the formation of two successive plastic hinges is elastic. A pictorial representation of this strategy for two incremental steps on a force-displacement diagram may be seen in Fig.2.
Various examples of application will be presented.

REFERENCES

[1] K.V. Spiliopoulos, "On the automation of the force method in the optimal plastic design of frames", *Comp. Meths. Appl. Mech. Engng.,* Vol. **141**, pp. 141-156, (1997).

[2] G. Maier, "Quadratic programming and theory of elastic-perfectly plastic structures", *Meccanica*, Vol. **3**, pp. 265-273, (1968).

[3] D.L. Smith, "The Wolfe – Markowitz algorithm for nonholonomic elastoplastic analysis", *Eng. Struct.*, Vol. **1**, pp.8-16, (1978).

[4] D. Goldfarb and A. Idnani, "A numerically stable dual method for solving strictly convex quadratic programs", *Math. Program.*, Vol. **27**, pp. 1-33, (1983).