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Fast Algorithms for Nonlinear Systems of Equations and Optimization

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ABSTRACT

Many large and hard, nonlinear, computational problems, such as porous medium flow, the design of structural systems [1], and tomography [2], lead to sequences of linear and nonlinear algebraic systems. The nonlinear systems can be solved by Newton or quasi-Newton methods, leading to an additional sequence of linear problems. The solution of such systems is typically the bottleneck in simulations and optimizations.

We will present a set of techniques to solve such problems efficiently. The key to efficient solvers is to update and reuse, hence *recycle*, the generated search spaces and preconditioners, and modify the Newton scheme such as to exploit this optimally. Recycling Krylov subspaces leads to drastic reductions in iterations and runtimes [1,2,3]. Updating preconditioners reduces the often high computational cost of computing the preconditioner while maintaining or even improving the convergence rate [4]. By modifying the (quasi-)Newton method we are able to make multiple nonlinear iterations with a very modest number of (preconditioned) matrix-vector products in the linear solver, sometimes none [5]. In experimental results it also appears that the number of nonlinear steps where a new accurate Jacobian must be computed is reduced.

We will discuss these methods, put the various techniques into perspective with respect to applications, and demonstrate their use on large-scale problems.

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