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Preconditioning of Newton-GMRES solvers for Discontinuous Galerkin Problems

*** Per-Olof Persson[1] and Jaime Peraire[2]**

[1]Department of Mathematics and [2]Department of Aeronautics and Astronautics
Massachusetts Institute of Technology
77 Massachusetts Avenue, Cambridge, MA 02139, U.S.A.
[1]persson@mit.edu and [2]peraire@mit.edu, http://3dg.mit.edu

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ABSTRACT

A fundamental problem with Discontinuous Galerkin methods is their high computational and storage cost. This is partly because they require more degrees of freedom than other methods, but mainly due to the wide nodal stencils which result in very large Jacobian matrices in implicit solvers. In this work we address the high cost of solving the corresponding linear systems of equations and propose a new preconditioner for implicit solution of stationary or time dependent Discontinuous Galerkin problems [1]. The viscous terms are discretized using the Compact Discontinuous Galerkin (CDG) method [2], but the results should be representative for other schemes as well. We consider several existing preconditioners such as block-Jacobi and Gauss-Seidel combined with multi-level schemes which have been developed and tested for specific applications. While our results are consistent with the claims reported, we find that these preconditioners lack robustness when used in more challenging situations involving low Mach numbers, stretched grids or high Reynolds number turbulent flows. We propose a preconditioner based on a coarse scale correction with post-smoothing based on a block incomplete LU factorization with zero fill-in (ILU0) of the Jacobian matrix. Our block-Minimum Discarded Fill algorithm numbers the elements such that the error in the ILU0 factorization is minimized in a greedy fashion, a step which turns out to be critical for convection dominated problems.

While little can be said in the way of theoretical results, the proposed preconditioner is shown to perform remarkably well for a broad range of representative test problems. These include compressible flows ranging from very low Reynolds numbers to fully turbulent flows using the Reynolds Averaged Navier Stokes equations discretized on highly stretched grids.

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