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ON MULTI-SCALE/MULTI-GRID FE ANALYSIS OF HETEROGENEOUS QUASI-BRITTLE MATERIALS

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ABSTRACT

Multi-scale analysis techniques aim to capture the macroscopic behaviour, that is rooted in the underlying heterogeneous microstructure, via the consistent and detailed modeling of the microstructure. Computational homogenization [1,2] is one such technique which utilizes nested multi-level finite element analsyses, with discretisations on both the micro and macro levels. However, such a technique is only valid where the principle of separation of scales is valid (i.e. large differences between the micro and macro-scales). In situations where there is poor scale separation, a single micro-scale analysis is required [3]. Miehe [3] has shown that geometric multigrid techniques are suitable for solving such large scale problems and that these may be considered numerical multiscale approaches. In the case of both good or poor scale separation, the transfer of information between scales (scale bridging) is a vital part of the numerical procedure. However, the multigrid approach needs to be generalised to solve problems for softening materials, in order to avoid mesh dependency.

Initially, this paper focuses on the meso-scale analysis of quasi-brittle materials using a corotational hybrid-Trefftz formulation [4] for modelling cohesive cracks. The formulation is characterised by the fact that stresses are approximated within the domain of the element and the stiffness can be expressed via a boundary rather than a domain integral. Thus, compared to their FEM counterpart, hybrid-Trefftz stress elements exhibit faster convergence of the stress fields. Furthermore, the displacements are approximated on the boundary of each element and the displacement basis is defined independently on each element interface. Thus, the overall bandwidth of the stiffness matrix is very small and computationally efficient to solve. A corotational formulation [5] for hybrid-Trefftz elements is also introduced in order to capture the effect of geometric nonlinearities associated with moderate rotations. The model's performance is demonstrated, illustrating crack propagation and the influence of geometric nonlinearities.

This paper builds on the geometric multigrid technique of Miehe [3] to develop a technique suitable for the multi-scale modelling of quasi-brittle materials with poor scale separation. This technique utilizes a two grid approach – a coarse macro-scale grid

and a fine meso-scale grid. On the macro-scale a higher-order micro-polar homogeneous continuum is assumed; on the meso-scale the individual phases of the material are explicitly modelled within the framework of a classical continuum, utilising the hybrid-Trefftz stress elements described earlier. This paper will focus on the scale bridging and numerical implementation of this technique and will be demonstrated using a number of numerical examples.

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