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The hybrid wavelet-finite element method of arbitrary lines for environmental problems: study of convection-diffusion and numerical convergence

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ABSTRACT

The method of arbitrary lines (**mal**)[1-4] constitutes a general hybrid approximation methodology for solving problems of which the solution exhibits strong anisotropic character, e.g. thin layers in the convection-dominated convection-diffusion processes in fluid mechanics and environmental problems; shear bands in finite strain visco-plasticity; and in some problems of composite materials containing straight or tubular fibers. In the **mal** we semi-discretize the given partial differential equation(s) (PDE) in variational form and obtain an approximate system of boundary value problems for ordinary differential equations (BVP-ODE) along curvilinear lines, that we then solve efficiently by state-of-the-art (e.g. wavelet based) ODE solvers. In this way, the **mal** offers a general platform to approximate the solution in a hybrid that to closely reflect the anisotropic structure of the solution.

In this paper we numerically examine the convergence of the **mal** hybrid approximation for the convection-diffusion equation in the convection-dominated cases (i.e. $\varepsilon/|\beta| \ll 1$, where ε is the diffusion coefficient and β is the given velocity). To account for the one-dimensional nature of the high gradients (rapid variation in one direction and smooth in the others), the **mal**-approximation is constructed by the hp-FEM (for the regular part along **mal**-*faces*) hybridized with the multiscale wavelet approximation (for the part of lower regularity along **mal**-*lines*). Thus, the total error involved in the **mal**-solution contains two parts: one from the hp-FEM and the other from the wavelet approximation. Here, we first define a so-called *face-wise maximum energy norm* as the error measure, which measures the error from FEM (along *faces*) in a global weak sense, and the error from wavelet collocation ODE-solver (along *lines*) in a discrete manner. We then study numerically the **mal** performance for two test problems involving the thin layers arising in the convection-dominated case. In particular we examine the convergence behavior of (i) the semi-discretization by plotting for various values of ε the error vs. the number of **mal**-ODEs as the polynomial degree increases (for a fixed element), (ii) the

wavelet-ODE solver by plotting the error vs. the total number of collocation points.

Numerical results show that when *lines* are positioned approximately normal to the layers (see [3] sect.4) the rate of convergence of **mal** semi-discretization depends only on the smoothness of the true solution in the direction transverse to the **mal**-*lines*. With the hp-FEM approximation along *faces*, it converges exponentially. Thus **mal** generally offers special advantages for problems whose solution in one direction differs substantially from that in the other (almost normal) direction. For layer problems possessing essentially one-dimensional structure the rates of convergence are ε-uniform. Moreover, along **mal**-*lines*, due to the space refinement property of the multiscale wavelet used (see e.g. [5]), the grid on each **mal**-*line* is naturally adapted independently of the rest – rendering an efficient **mal**-ODE solver for capturing steep layers.

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