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IMPLICIT AND SEMI-IMPLICIT DISCONTINUOUS GALERKIN SCHEMES FOR UNSTEADY INCOMPRESSIBLE FLOWS

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ABSTRACT

In the recent past several high-order discontinuous Galerkin (DG) methods for incompressible viscous flows have been actively developed and analyzed in a number of papers, [1–5]. In this paper we aim at giving a contribution in this field by comparing two time integration schemes for the high-order DG discretization of unsteady, inviscid and viscous, incompressible flows.

The high-order implicit DG method here employed has been introduced in [6] for the incompressible Stokes, Oseen and Navier-Stokes equations and extended in [7] and [8] to deal with steady and unsteady problems of inviscid, viscous and natural convection incompressible flows. The method is fully implicit and applies to the governing equations in primitive variable form. Its distinguishing feature is the formulation of the inviscid interface flux which is based on the solution of local Riemann problems associated with the artificial compressibility perturbation of the Euler equations.

Even if the implicit method proved to be efficient, accurate and robust in computing several well documented benchmark problems covering a wide range of Reynolds numbers, there are applications where time scales of interest can be very short. Such cases are those where violent transients due to the very high (theoretically infinite) velocity of propagation of pressure waves is the physical phenomenon of primary interest. The implicit method can be applied to these cases as well, but also lighter semi-implicit methods can be worth being considered.

The semi-implicit DG method here proposed shares the same formulation of inviscid and viscous fluxes of our implicit method, enforces continuity implicitly and defines implicitly the pressure gradient in the momentum equation. Instead, the inviscid and viscous fluxes in the momentum equation are treated explicitly. From this formulation it is possible to arrive at a single implicit equation for the pressure variation which is characterized by a compact stencil size. Upon solving for the pressure variation (either with a direct or an iterative linear solver) we are finally left with the time variations of pressure and velocity over the time step. These are then combined in suitable multistage Runge-Kutta schemes to increase time accuracy. Cheaper alternatives to Runge-Kutta methods, allowing to solve the implicit equation for the pressure variation only once per time step, could be fruitfully adopted.

The semi-implicit method will be applied to compute the impulsively started lid driven cavity flow at various Reynolds numbers and the inviscid "thin" double shear layer problem. The accuracy of the semi-implicit method will be assessed by comparison with the results of the fully implicit DG method. All the numerical tests will be computed up to $\mathbb{P}_6$ polynomial approximation.

REFERENCES

[1] J.-G. Liu and C.-W. Shu. "A high-order discontinuous Galerkin method for 2D incompressible flows". *J. Comput. Phys.*, Vol. **160 (2)**, 577-596, 2000.

[2] B. Cockburn, G. Kanschat, D. Schotzau, C. Schwab. "Local discontinuous Galerkin methods for the Stokes system". *SIAM J. Numer. Anal.*, Vol. **40 (1)**, 319-343, 2002.

[3] B. Cockburn, G. Kanschat, D. Schotzau. "The local discontinuous Galerkin method for the Oseen equations". *Math. Comp.*, Vol. **73 (246)**, 569-593, 2003.

[4] B. Cockburn, G. Kanschat, D. Schotzau. "A locally conservative LDG method for the incompressible NavierStokes equations". *Math. Comp.*, Vol. **74 (251)**, 1067-1095, 2005.

[5] K. Shahbazi, P. F. Fischer, C. Ross Ethier. "A high-order discontinuous Galerkin method for the unsteady incompressible NavierStokes equations". *J. Comput. Phys.*, Vol. **222**, 391–407, 2007.

[6] F. Bassi, A. Crivellini, D. A. Di Pietro, S. Rebay. "An artificial compressibility flux for the discontinuous Galerkin solution of the incompressible Navier-Stokes equations". *J. Comput. Phys.*, Vol. **218 (2)**, 794–815, 2006.

[7] F. Bassi and A. Crivellini. "A high-order discontinuous Galerkin method for natural convection problems". In P. Wesseling, E. Oñate, J. Périaux (eds.), Electronic Proceedings of the ECCOMAS CFD 2006 Conference, Egmond aan Zee, The Netherlands, 5-8 September 2006.

[8] F. Bassi, A. Crivellini, D. A. Di Pietro, S. Rebay. "An implicit high-order discontinuous Galerkin method for steady and unsteady incompressible flows". *Comput. & Fluids*, Vol. **36**, 1529–1546, 2007.