8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

Model Normalization for the Analysis of Inertial Parameters in Multibody Dynamics

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Key Words: *Normalization, inertial parameters, multibody systems, parameter estimation, sensitivity analysis.*

ABSTRACT

The understanding of the influence of inertial parameters on the dynamics behaviour and performance is of great importance in design, parameter estimation, and analysis in general. The dynamics model of a mechanical system can often be established so that it is linear in the inertial parameters that are to be studied. Therefore, the equations can be rearranged to result in the so-called *dynamics identification model* associated with a certain trajectory or range of motion. We consider a system with f degrees of freedom, where we are interested in the analysis of r inertial parameters in which the equations are linear. The trajectory is represented by h measurement points equally spaced in time. Then, the dynamics identification equations can generally be written as

$$\mathbf{A}\mathbf{x} = \boldsymbol{\tau} \tag{1}$$

where $\mathbf{A}$ is the *regression matrix* of dimension $hf \times r$, whose elements are all known quantities (functions of configuration, velocity, and acceleration), $\boldsymbol{\tau}$ is the $hf \times 1$ vector of applied forces and/or torques determined at the measurement points, and $\mathbf{x}$ is the $r \times 1$ vector of inertial parameters. This model along with methods of matrix computations can be used to study the effects of inertial parameters. A fundamental problem, however, is that the inertial parameters are generally not homogeneous in physical units. Therefore, matrix $\mathbf{A}$ also contains elements with different units. This can lead to meaningless analysis results if numerical procedures are "blindly" applied to $\mathbf{A}$. In this work, we address this issue and the normalization of the identification model of multibody systems for parameter estimation and sensitivity analyses. We will first deal with systems where the independent generalized coordinates are all of the same kind (e.g. rotational or translational). Therefore, the right-hand side of (1), $\boldsymbol{\tau}$, is homogeneous in units. It contains forces or torques only.

Normalization is of course possible in ad-hoc ways by selecting various weighting terms usually motivated by certain features of actuators, etc. Here we propose an alternative procedure, which is not an ad-hoc method. It exploits the underlying physical structure of the system. A key idea relates to the

formation of eigenvalue problems that are associated with the block partitioned form of the identification model. Each block contains one type of parameter (e.g. moments of inertia) and also associated with one type of generalized coordinate only (e.g. rotational or translational coordinate). The eigenvalue problems studied characterize the geometric representation of the block partitioned form of the dynamic model in terms of the inertial parameters.

In an eigenvalue problem with physical units, the units are automatically transferred to the eigenvectors. This gives rise to the possibility to introduce the concept of so-called *dimensionless inertial parameters*, $\boldsymbol{\delta}$, that are related to the actual parameters, $\mathbf{x}$[1], via

$$\mathbf{x} = \mathbf{S}\boldsymbol{\delta} \tag{2}$$

where $\mathbf{S}$ is an orthogonal matrix and its columns represent the base vectors of the *natural coordinate system* for the overall parameter space. These base vectors can be interpreted using the solution of the eigenvalue problems. Some components of the base vectors carry physical units, some others don't. Based on this the dynamics identification model can be expressed in terms of the dimensionless inertial parameters as

$$\boldsymbol{\Gamma}\boldsymbol{\delta} = \boldsymbol{\tau} \tag{3}$$

where $\boldsymbol{\Gamma} = \mathbf{CS}$ is a new, normalized regression matrix. This form expresses the dynamics in terms of dimensionless parameters. We will elaborate on the development of this model in detail in the presentation.

This approach leads to a formulation where the mathematical manipulations of arrays are in complete agreement with the physical units of the elements. This formulation makes it possible to introduce a set of essential inertial parameters in a physically consistent way. These can be used to characterize the dynamics of the system, and also to perform sensitivity studies in terms of the inertial parameters. An experimental, six degree of freedom dual-pantograph system is used to illustrate these possibilities.

[1] It is assumed that the structure of $\mathbf{x}$ is already such that the same kind of parameters are grouped together.