**8th. World Congress on Computational Mechanics (WCCM8)**
**5th. European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)**
**June 30 – July 5, 2008**
**Venice, Italy**

# Shape sensitivity analysis and Optimization of Structures using Isogeometric Approach

*** Seonho Cho[1], Seung-Hyun Ha[2]**

[1] Department of Naval Architecture and Ocean Engineering, Seoul National University,
Kwanak-Ro 599, Sillim-9-Dong, Kwanak-Gu,
Seoul, 151-742, Korea
secho@snu.ac.kr

[2] Department of Naval Architecture and Ocean Engineering, Seoul National University,
Kwanak-Ro 599, Sillim-9-Dong, Kwanak-Gu,
Seoul, 151-742, Korea
hash00@snu.ac.kr

**Key Words:** *Shape sensitivity analysis, Incompressible fluid, SPH projection method.*

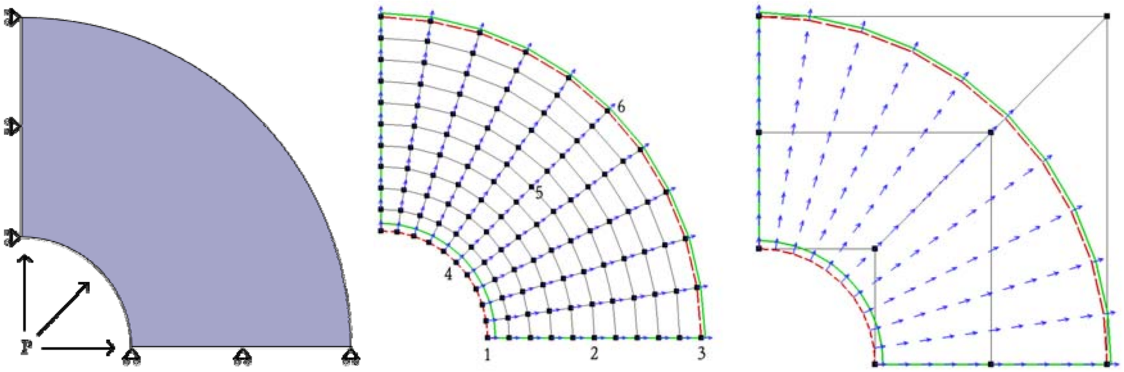
## ABSTRACT

Typically in finite element based engineering analysis, designs are embedded in computer aided design (CAD) systems and the finite element meshes are generated from the CAD data. The geometric approximation inherent in the mesh may lead to accuracy problems in response analysis and more adversely in design sensitivity analysis. Piecewise linear approximations of geometry are the root cause. Even though a mesh is constructed, further refinement requires tedious communication with the CAD system during design iterations. Isogeometric analysis concept based on NURBS (Non-Uniform Rational B-Splines) is developed by Hughes *et al.* [1]. The objective of isogeometric analysis is to develop an analysis framework, employing the same basis functions as used in the CAD systems and thus embedding exact geometry. The advantages of isogeometric approach are two folds: *firstly*, the geometric flexibility of the NURBS basis allows for the exact representation of geometry. *Secondly*, subsequent refinement does not require any further communication with the CAD systems so that mesh refinement procedures are significantly simplified.

A continuum-based adjoint sensitivity analysis method using the isogeometric approach is developed for elasticity problems. To obtain precise shape sensitivity, correct normal and curvature information should be taken into account in shape sensitivity expressions. However, in conventional finite element methods using linear interpolation functions, the normal and curvature information is generally inaccurate or missing due to lack of inter-element continuity of design space. In isogeometric approach, the basis functions generated from NURBS are directly used to construct an exact geometric model in response and shape sensitivity analyses. Refinements and design changes are easily implemented within the isogeometric framework, which maintains exact geometry without subsequent communication with a CAD description. In addition to the benefits of isogeometric analysis, the isogeometric design sensitivity analysis has the following advantages [2]: *firstly*, it provides more accurate sensitivity of complex geometries including higher order effects such as normal and curvature information. For instance, consider the quarter model of a plane elasticity problem subjected to internal pressure P as shown in Figure 1. For a design perturbation in radial direction, the analytical sensitivity of displacement is compared with finite differencing.

In Table 1, the column **C** denotes the agreement between the columns **A** and **B** using finite element approach. Since the linear elements are used in the sensitivity analysis, the normal vectors and curvature are intrinsically inaccurate or even missing and consequently the agreement is quite bad. Using the isogeometric approach excluding the normal and curvature term, the agreement (Column **F**) is not satisfactory as expected. However, when the normal and curvature information are included, excellent agreement is observed in Column **H**. *Secondly*, it vastly simplify the design modification of complex geometries without communicate with the CAD description of geometry during optimization process. Since the NURBS basic functions are used in isogeometric response and sensitivity analyses, design modifications are easily obtainable using the adjustment of control points which represent the geometric model.

We present some demonstrative numerical examples, where the accuracy and efficiency of the isogeometric sensitivity results are compared to the finite difference ones. Also, some examples for shape design optimization are demonstrated to verify the applicability and effectiveness of the proposed method.



(a) Problem description (b) Finite element mesh (d) Isogeometric control net

**Figure 1** Finite element and isogeometric models for shape sensitivity analysis

**Table 1** Higher order effects in displacement sensitivity

| DOF | A | B | C (%) | D | E | F (%) | G | H (%) |
|---|---|---|---|---|---|---|---|---|
| 1x | 9.17943E-5 | 1.84459E-5 | **497.64** | 9.42135E-05 | 2.30892E-05 | **408.04** | 9.41939E-05 | **100.02** |
| 2x | 7.46584E-5 | 3.16836E-5 | **235.64** | 7.02781E-05 | 3.86344E-05 | **181.91** | 7.02476E-05 | **100.04** |
| 3x | 6.54527E-5 | 2.98590E-5 | **219.21** | 6.57607E-05 | 3.09113E-05 | **212.74** | 6.57335E-05 | **100.04** |
| 4x | 6.49146E-5 | 1.30458E-5 | **497.59** | 9.40964E-05 | 2.30592E-05 | **408.06** | 9.40768E-05 | **100.02** |
| 5x | 5.28442E-5 | 2.24067E-5 | **235.84** | 7.01654E-05 | 3.85767E-05 | **181.89** | 7.01349E-05 | **100.04** |
| 6x | 4.62902E-5 | 2.11184E-5 | **219.19** | 6.56583E-05 | 3.08643E-05 | **212.73** | 6.56312E-05 | **100.04** |

**A**: finite difference sensitivity using finite elements
**B**: finite element analytical sensitivity excluding κ term
**C**: agreement between **A** and **B**
**D**: finite difference sensitivity using isogeometric analysis
**E**: isogeometric analytical sensitivity excluding κ term
**F**: agreement between **D** and **E**
**G**: isogeometric analytical sensitivity including κ term
**H**: agreement between **D** and **G**

## REFERENCES

[1] T.J.R. Hughes, J.A. Cottrell, Y. Bazilevs, Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement, *Computer Methods in Applied Mechanics and Engineering* **194** (2005) 4135-4195.

[2] S. Cho, S.H. Ha, Shape design optimization of geometrically nonlinear structures using isogeometric analysis, *9th United States National Congress on Computational Mechanics*, San Francisco, California, U.S.A., July 22-26, 2007.