8th. World Congress on Computational Mechanics (WCCM8)
5th. European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 – July 5, 2008
Venice, Italy

Shape sensitivity analysis for incompressible fluid using SPH projection method

***Yoondo Ha[1], Seonho Cho[2]**

[1] Department of Naval Architecture and Ocean Engineering, Seoul National University, Kwanak-Ro 599, Sillim-9-Dong, Kwanak-Gu, Seoul, 151-742, Korea
naval98@snu.ac.kr

[2] Department of Naval Architecture and Ocean Engineering, Seoul National University, Kwanak-Ro 599, Sillim-9-Dong, Kwanak-Gu, Seoul, 151-742, Korea
secho@snu.ac.kr

Key Words: *Shape sensitivity analysis, Incompressible fluid, SPH projection method.*

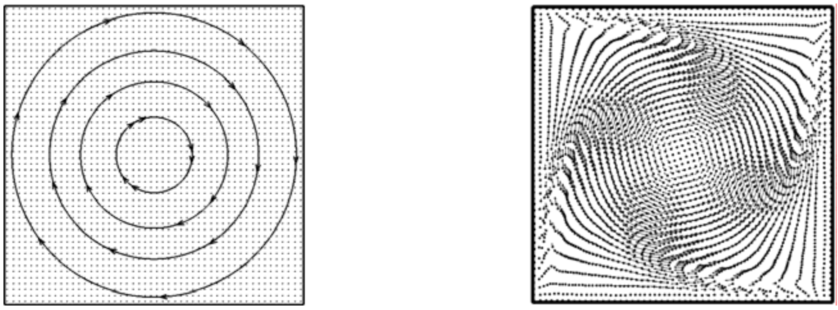
ABSTRACT

An analytical shape design sensitivity analysis (DSA) method for time-dependent incompressible Newtonian fluid is developed using the direct differentiation approach [1]. Governing equations for the incompressible fluid are the continuity equation and the equations of motion. In conventional smoothed particle hydrodynamics (SPH) method to simulate incompressible flows, the pressure is approximately obtained from the state of equation using artificially small sound speed. The incompressibility can not be satisfied strictly and the time steps for the integration of governing equations should be very small. Thus, unstable responses are observed around the fluid boundary, which have significantly adverse impacts on the computation of shape design sensitivity.

To alleviate this difficulty in the SPH method, a pressure projection is employed to enforce the incompressibility [2]. Without imposing incompressibility, velocity field is firstly computed by integrating the equations of motion at an intermediate step. Then, the incompressibility is enforced in this intermediate velocity field by solving a pressure Poisson equation derived from an approximate pressure projection. Even though the additional pressure Poisson equation should be solved, comparing with the conventional SPH method, total computational cost for the SPH projection method is not expensive because larger time steps can be used. The computational cost for the sensitivity analysis is also trivial comparing with that for the response analysis since the factorized system matrix for the pressure Poisson equation is readily available through the response analysis and can be directly utilized in the computation of analytical sensitivity.

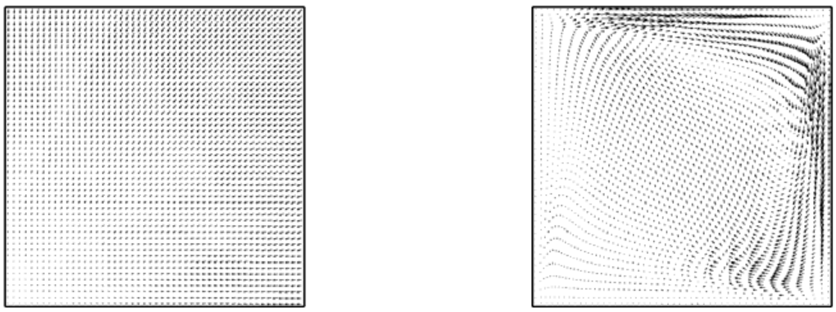
For instance, consider the two-dimensional vortex spin-down simulation shown in Figure 1-(a), in which the circles denote the velocity streamlines. A uniform time step of 0.005 is used for the SPH projection method and 0.001 for the conventional SPH method, which is based on the CFL stability constraints [2]. To compare the computational efficiency of the methods, the simulations are performed until the time step $t = 5$. The position and velocity vectors of particles at the time step $t = 5$ using the SPH projection method are illustrated in Figure 1-(b), where no unstable particles around the boundary are observed. The computing cost required for the SPH projection method is 830.07 seconds whereas for the conventional SPH method 1099.69 seconds.

For the shape variation that the upper right corner of the fluid domain is slightly perturbed, shape DSA is performed. The shape sensitivity of velocity for each particle at time step $t = 2$ is depicted in Figure 2. The sensitivity results are physically interpretable considering the shape changes. Also, the efficiency of the analytical sensitivity is compared with the finite difference sensitivity in Table 2. As increasing the number of design variables, the efficiency is significantly enhanced. It just takes around 15 % of the finite differencing so that the proposed shape DSA method is practically applicable for the shape design optimization of incompressible fluid problems.



(a) Initial velocity streamline (b) Position and velocities at $t = 5$

Figure 1 Vortex spin-down simulation by SPH projection method



(a) Initial shape velocity (b) Shape sensitivity of velocity at $t = 2$

Figure 2 Shape sensitivity result for vortex spin-down simulation

Table 2 Computation cost (seconds)

Number of design variables	Analytical sensitivity (A)	Finite difference sensitivity (B)	A/B Ratio (%)
1	1,029.35	1,660.14	62.00
20	4,815.67	33,202.8	14.50
400	80,542.07	664,056	12.13

REFERENCES

[1] E.J. Haug, K.K. Choi, and V. Komkov, *Design sensitivity analysis of structural systems*, Academic press, INC., 1986.

[2] S.J. Cummins and M. Rudman, "An SPH Projection method," *Journal of Computational Physics*, Vol. **152**, pp. 584–607, (1999).