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# A UNIFIED DISCRETIZATION APPROACH FOR SHEAR-RIGID AND SHEAR-DEFORMABLE SHELLS

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## ABSTRACT

We present a unified finite-deformation kinematic model and an associated discretization scheme for thin (shear-rigid) and thick (shear-flexible) shells. The kinematic model is an extension of the Kirchhoff-Love type model previously introduced in [2] and is of the form

$$\varphi = x + \lambda\theta^3(n + w) \tag{1}$$

where $\varphi$ is the position vector of a material point in the deformed configuration, $x$ is the deformed shell midsurface, $n$ is the unit normal to the deformed midsurface and $w$ is an additional vector for allowing out-of-plane shear deformations. The parameter $\theta^3$ determines the position of a material point on the normal $n$ and the thickness stretch $\lambda$ is the ratio between the reference and deformed thickness. Importantly, in Equation 1 the only independent variables are the vectors $x$ and $w$ and the scalar $\lambda$. Furthermore, the proposed kinematics converges for thin shells (thickness $\rightarrow 0$) towards a Kirchhoff-Love type model with $|w| \rightarrow 0$.

The conforming finite element discretization of the shell energy functional based on Equation 1 requires $C^1$-continuous shape functions. In the developed method, smooth subdivision shape functions are used for interpolating the mid-surface position vector $x$ and the shear vector $w$ [1]. In the thin-shell limit, the resulting finite elements do not exhibit shear locking simply because the interpolation of $x \neq 0$ and $w \equiv 0$ does not lead to any compatibility problems. In addition, no rotational degrees-of-freedom are used, which facilitates the implementation for the finite deformation case.

To study the performance of the developed shell elements, we computed, amongst others, pressure loaded clamped square plates with length to thickness ratios of $L/t = 5$, $L/t = 7$, $L/t = 2000$ and $L/t = 10000$. As shown in Figure 1, for moderately thick plates with $L/t = 5$ and $L/t = 7$, the difference between the shear-flexible and shear-rigid analytic solutions is in the order of $25\%$. Therefore, for such problems shell models, which account for the out-of-plane shear deformations have to be used. As evident from Figure 1 the results of the proposed subdivision shell elements are in excellent agreement with the analytic solution. Furthermore, the developed elements perform similarly well for extremely thin plates with $L/t = 2000$ and $L/t = 10000$ (see Figure 2).

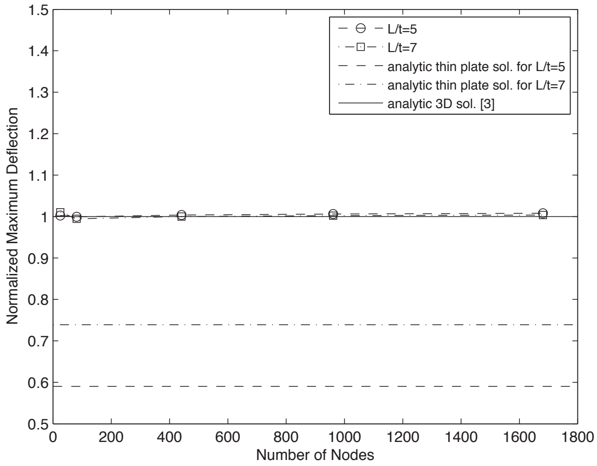


Figure 1: Convergence of normalized maximum deflections for two clamped moderately thick plates with length to thickness ratios of 5 and 7.

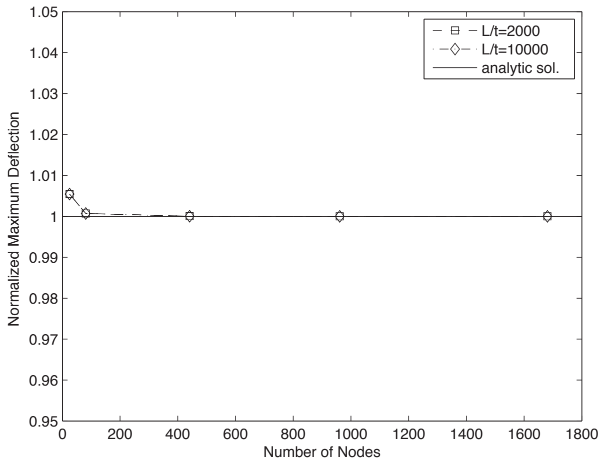


Figure 2: Convergence of normalized maximum deflections for two clamped thin plates with length to thickness ratios of 2000 and 10000.

In summary, subdivision shell elements are equally suitable for analysing moderately thick and thin shell problems. In particular, for problems where several length scales are simultaneously present, such as in wrinkling, buckling or shear banding of large structures, the proposed extended kinematic model enables to resolve all scales with similar fidelity.

## REFERENCES

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