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MODELLING OF FAILURE IN SOLIDS BY EMBEDDED DISCONTINUITIES. DISPLACEMENTS AND STRAIN-DISPLACEMENTS FORMULATIONS OF FINITE ELEMENT

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ABSTRACT

The problem of strain localization in a continuous solid, which eventually leads to the failure of a structure, is studied in this paper using the discrete approximation of the embedded discontinuities approach (**EDA)**.

A discrete constitutive model is used to simulate the mode I of damage that the solid undergoes through a history of loading. This constitutive model corresponds to an anisotropic local model where the information of the parameters, required to simulate the damage, comes only from the discontinuity and not from the whole solid [1, 2].

For the approximation of the problem two different finite element formulations are studied: a displacement formulation, where the only independent field is that of displacements and the symmetry of the stiffness matrix is not lost during the analysis process; and the mixed strain-displacement (**MSD**) formulation, variation of Hellinger-Reissner, which has as independent fields, strains and displacements, for which the symmetry of the stiffness matrix is not guaranteed when an element undergoes localization[3, 4].

The variational finite element formulations investigated in this paper (displacements and **MSD**) are correspondingly stated as:

$$\Pi(u, [\![u]\!]) = \int_{\Omega/S} [W(\varepsilon^u) - \mathbf{b}\cdot u]\, d\Omega - \int_{\Gamma_\sigma} \bar{\mathbf{t}}\cdot u\, d\Gamma + \int_S T\cdot [\![u]\!]\, dS$$

$$\Pi(u, \bar{\varepsilon}, [\![u]\!]) = \int_{\Omega/S} [\sigma^\varepsilon(\bar{\varepsilon}^u - \bar{\varepsilon}) + W(\bar{\varepsilon}) - \mathbf{b}\cdot u]\, d\Omega - \int_{\Gamma_\sigma} \bar{\mathbf{t}}\cdot u\, d\Gamma + \int_S T\cdot [\![u]\!]\, dS$$

where $W(\varepsilon^u)$ is the strain energy function of the strains derived from the displacement field, u, the body forces are denoted by $\mathbf{b}$, the tractions imposed over the surface are $\bar{\mathbf{t}}$, the elastic tensor at the discontituity is T, the displacement jump is $[\![u]\!]$, σ^ϵ corresponds to the stresses dependent of the strain field, $\bar{\varepsilon}^u$ are the regular strains dependent of the displacements, $\bar{\varepsilon}$ is the regular independent strain field and $W(\bar{\varepsilon})$ is the strain energy function of the independent strain field. Since the **MSD** variational formulation

considers the strains field as independent, its variation in a homogenous continuum is smooth.

The finite element approximation of the **MSD** formulation where the strain field is continuous across the boundaries of the elements, figure 1, is a better approximation than the displacement formulation particularly when the direction of propagation of a discontinuity through the domain of each element, which undergoes localization, needs to be determined. With a displacement formulation the strain and stress fields are discontinuous, the approximation of tracking of a discontinuity requires an additional stress recovery technique to obtain a continuous stresses field in equilibrium.

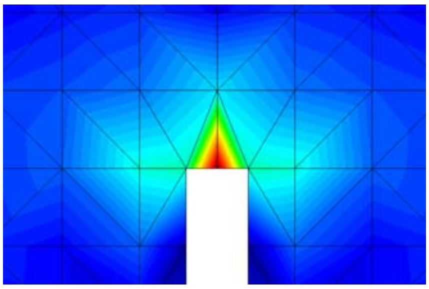

Figure 1. Continuity of strains in the solid with the mixed Reissner formulation.

The approach of embedded discontinuities, presented in this paper together with the adopted mixed finite element approximation, guarantees the continuity of strains and hence of stresses in the domain of the continuum. This fact eliminates the necessity of a tracking approximation such as a stress recovery technique for smoothing the stresses field in and around the tip of the discontinuity.

Relative advantages and drawbacks, of the displacements and **MSD** finite element formulations, are discussed. Considering the importance of Babuska-Brezzi condition, the validation of the proposed **MSD** formulation, is shown by demonstrating that this condition is satisfied.

To validate and illustrate the investigated finite element formulations, numerical examples are presented showing their correctness, capabilities and limitations. Finally important conclusions are discussed around these models of finite elements relatives to their variational formulations and numerical implementations.

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