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ABSORBING BOUNDARY CONDITIONS AND THE PERFECTLY MATCHED LAYER MODEL IN ELASTIC WAVE PROPAGATION ANALYSIS

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ABSTRACT

The investigation of unbounded domains is often of interest in various fields of science and engineering. A typical example can be found in structural dynamics, where the elastodynamic wave equation for an unbounded domain needs to be solved in order to describe the dynamic interaction of a structure and its underlying soil. In particular, the investigation of foundation vibration problems or of earthquake ground motion is a rather challenging task.

The definition of an unbounded domain requires the enforcement of a radiation condition in any unbounded direction. Irregularities in the geometry of the domain, or in the material, often compel a numerical solution of the problem using different mathematical formulations, e.g. a coupling of the finite element method (FEM) and the boundary element method (BEM). One alternative, however, is an approach using only the FEM in combination with an artificial boundary taking care of the absorption of waves travelling to infinity. This so-called absorbing boundary imposes the necessary radiation condition and ensures no spurious reflections back into the domain of interest.

Among the various methods and techniques for imposing the Absorbing Boundary Condition (ABC), probably the simplest one is the viscous damping method [3]. During the last 40 years several new techniques emerged solving the problem of the unbounded domain and its radiation conditions more or less successfully with certain advantages but also disadvantages. In the last few years the usage of so-called Perfectly Matched Layers [1] turned out to be very promising since the results are very accurate and the approach is rather efficient.

A perfectly matched layer is an absorbing boundary layer for linear wave equations. It absorbs, almost perfectly, propagating waves of all non-tangential angles of incidence

and of all non-zero frequencies. In general, the boundary can be defined with the same material as the analyzed domain, but having attenuation characteristics that damp the outgoing and reflected wave within the layer thickness. Thus, wave propagation in an unbounded domain can be modelled through a bounded domain surrounded by a suitably defined PML.

The current contribution deals with the development of a PML formulation to be applied as absorbing boundary condition for the elastic wave equation in time-harmonic elastodynamics [2]. Although the PML allows using different numerical techniques to be combined with, here a combination with the FEM is suggested.

To demonstrate the applicability and accuracy of the new approach, two examples are discussed:

First, a comparative study of a rather simple model is performed. The investigated system is representative for a longitudinal wave propagation problem in an unrestricted rod on an elastic foundation, Fig.1. The results demonstrate the remarkable features of the PML method as well as its accuracy when compared to the exact analytical solution.

Second, the PML concept is applied to a two-dimensional plane-strain problem of rigid footing over visco-elastic halfspace [4]. Once again, the excellent efficiency of the new approach is demonstrated when looking at the vibrations of a surface foundation under time-harmonic excitation, Fig.2. Moreover, the stability issues of the solution are addressed and compared to classical absorbing boundary conditions.

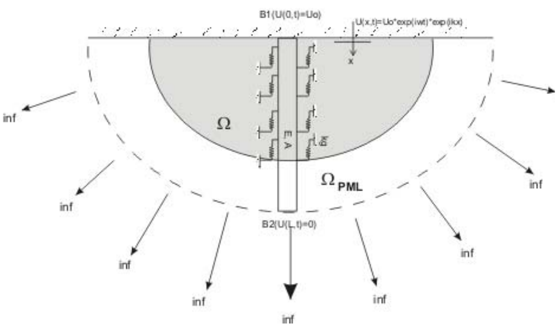

Fig.1 Model of semi-infinite rod on (visco-) elastic foundation

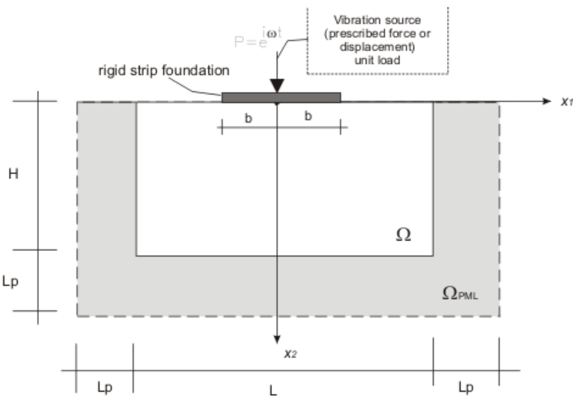

Fig.2 Model of Rigid Surface footing over (visco-) elastic halfspace

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