High order dispersive homogenization of multidimensional periodic materials by mode decomposition

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Unlike in the static case, periodic materials under dynamic conditions need a highly refined consideration of the microstructure topology in order to generate a model for an equivalent homogeneous material. Consider, for example, the two periodic unit cell configurations shown in Figure 1. Under static conditions, the two unit cells are identical from a homogenization point of view. In contrast, each has a different set of macroscale dynamical properties, especially at high frequencies. This is attributed to the complex wave dispersion phenomenon that takes place across the material phase interfaces.

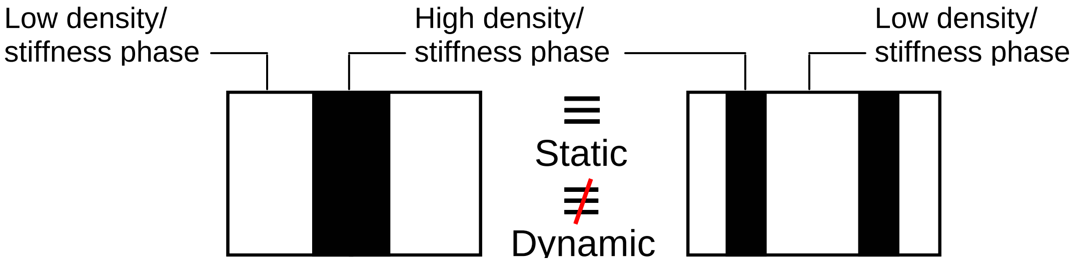

Fig. 1: Two unit cells (A at the left and B at the right) representative of periodic materials. These two designs are statically, but not dynamically, equivalent.

Figure 2 shows the dispersion curves for each of the unit cells shown in Figure 1, and also the curves corresponding to a statically homogenized media. Clearly, the static homogenization is only accurate at the long wavelength (low wavenumber) limit. In this work, a Multiscale Assumed Strain Projection method is used to predict the dispersion curves for periodic elastic media with good accuracy up to high frequency values covering several branches in the spectrum. Results for simple one-dimensional materials were presented previously[1]; the current works extends the method to multiple dimensions. A square unit cell composed of a square high density/stiffness phase inclusion centered within a low density/stiffness host material is considered, as shown in the inset of Figure 3. The dispersion curves for out-of-plane, SH polarization wave motion are shown in the figure; the black curves were obtained by direct application of the finite element (FE) method, and the blue curves were obtained using the proposed homogenization method with FE implementation. Excellent prediction accuracy is achieved at a notably small increase in computational cost.

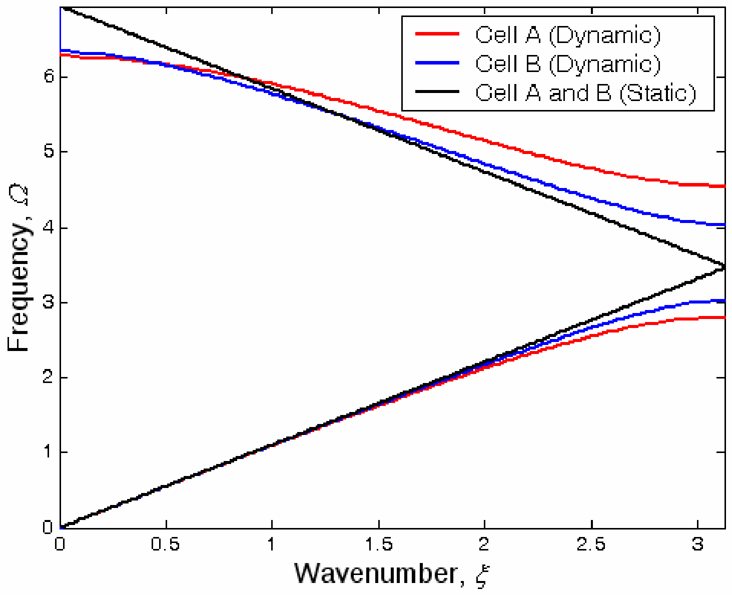

Fig. 2: First two branches of dispersion curves for unit cells A (red) and B (blue) and for equivalent homogenized material under static conditions (black).

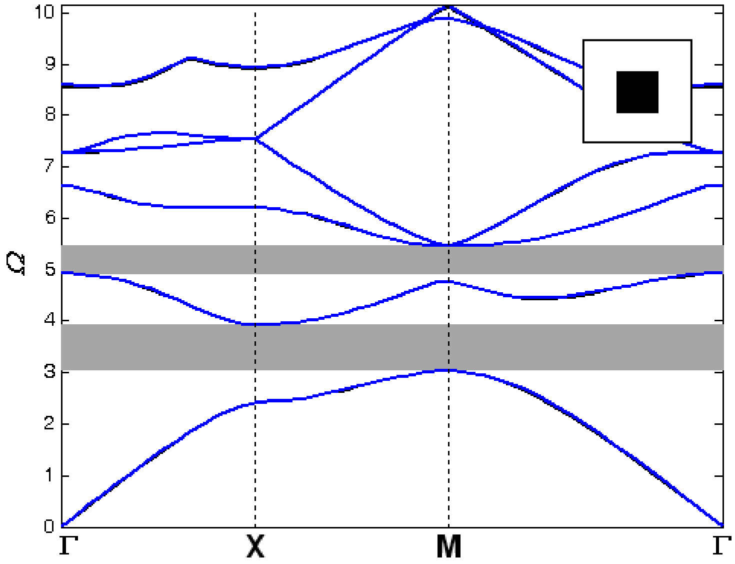

Fig. 3: Out-of-plane wave motion dispersion curves for a two-dimensional unit cell (shown in inset): homogenized FE solution (blue) and direct FE solution (black).

References

1. Hussein, M.I. and Hulbert, G.M., "Mode-Enriched Dispersion Models of Periodic Materials within a Multiscale Mixed Finite Element Framework," *Finite Elements in Analysis and Design*, **42**(7), 602-612, 2006.