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The Meshless Local Petrov-Galerkin method for the Stefan Problem in systems with complex geometry

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ABSTRACT

The heat transfer phenomena involving phase change either solidification or melting, appear frequently in nature and in industrial applications. In nature the melting and solidification processes are found in geophysical systems such as the growth of Earths's crust and the ice formation (or melting) in the Earths's polar regions. In industrial applications the phase change phenomena are present in the cast of pure metals and alloys, in mono-crystal growth techniques and in plastic extrusion manufacturing processes. The simulation of the phase change (liquid-solid) process has been widely studied by using mesh-based algorithms (finite difference, finite volume, spectral elements[1], etc.). These methods have provided satisfactory results, however the accuracy of the simulations is strongly dependent on the general characteristics of the meshes (connectivity and aspect ratio between the elements, among others). Additionally when the simulation of the phase change process is carried out, by using moving interface numerical techniques, the elements can be over-deformed, requiring a remeshing process. However the remeshing is frequently an expensive task. In this paper we present a Meshless Local Petrov Galerkin (MLPG) method for the solution of the phase change solid-liquid (Stefan) problem. This method is based on the weak formulation of the governing equations and the integration is performed over local sub-domains defined by particles (nodes) and their neighbours. The initial position of each particle in the domain, can be either randomly distributed or ordered (this last avoids a complicated search algorithm to find the neighbours of each particle). Among the advantages of the MLPG method are the following: (1) a global mesh is not needed and (2) the local sub-domains can be overlapped. Even though the MLPG method has been applied in different fields including fracture mechanics problems and some fluid dynamic applications [2,3,4], to the knowledge of the authors this approach has not been used to solve the multi-dimensional Stefan problem in systems with complex geometry. In this investigation, the meshfree (particle) MLPG approach has been used to simulate the melting process in confined systems without natural convection in the liquid phase. The weak form of the heat equations in the liquid and solid phases has been formulated in a cartesian coordinate system. The treatment of the interface is based on a local transformation technique (moving interface approach), which accurately tracks the position of the interface[1]. The local transformation of the independent variables in the heat equations,

leads to the generation of convection correction terms which have been included in the mathematical models. Hence we solve a system of two coupled convection-diffusion equations. The weak form of the energy balance equation which governs the velocity of the interface, has also been integrated by using the MLPG approach. The new position of the particles located at the interface, is calculated by an Euler forward scheme. The displacement vector of the particles that are located out of the interface is calculated by solving a Poison equation for each cartesian direction (pseudo-elastic solid deformation approach). Again each Poisson equation has been solved by the MLPG method. The shape functions needed to approximate the dependent variables (temperature, normal velocity of the interface and displacements of the particles) in a local domain have been generated by using the Moving Least Square (MLS) scheme. To integrate the whole set of equations and to differentiate the dependent variables in systems with complex geometry, we introduce a one-to-one isoparametric mapping between the physical and computational domains. The integration of the equations in the local computational domain is performed by using the Gauss-Lobatto-Legendre quadrature rule. The weight function used in the MLS scheme and in the weighted residual process, is a compact support fourth order spline. The temporal discretization of the modified heat equations is carried out by using the Crank-Nicolson scheme$^{(5)}$. The dirichlet and Neumann boundary conditions have been enforced by using the method proposed by Šterk $et.al$ (2005). Firstly we present the numerical simulation of three test cases which have analytical solution: (1) the melting process of a solid bar, (2) the melting process of the solid confined between concentric cylinders with aspect ratio 0.35 and (3) the melting process of the solid confined between concentric spheres with aspect ratio 0.35. In the three cases the Stefan number was fixed to St=10. In the test cases two and three, the melting process is from the internal boundary (cylinder or sphere). We have found that the obtained results are in full agreement with the analytical solutions. We also present two additional test cases: (4) the two dimensional melting process in concentric cylinders and (5) the three dimensional melting process in concentric spheres. For the three dimensional cases corresponding to the concentric spheres, the cubed sphere algorithm has been used to generate the initial location of the particles in the cartesian coordinate system. The resulting particle distribution is free from singularities at the poles of the spheres[6]. We conclude that the MLPG method (without a global mesh) coupled with the pseudo-elastic approach can be used as a reliable methodology for the solution of the melting process in systems where the natural convection of the liquid phase can be neglected.

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