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A new modelling methodology of MEMS based on Cosserat theory

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ABSTRACT

Advanced design and manufacturing methodologies involve a variety of software tools for MEMS design that deal with the analysis of complex geometrical structures and the assessment of various interactions among different energy domains and components. Besides, the MEMS market is growing very fast, but surprisingly, there is a lack of modelling and simulation methodology for precise performance verification of MEMS products in the nonlinear regime.

At the present time, modelling MEMS can be classified into two categories as shown in Figure 1 and is widely made using either Finite Element Analysis (FEA), component level modelling, Boundary-Element Method (BEM) or lumped level modelling. For accurate representation, the number of equations increases significantly and the model tends to be cumbersome and complicated, preventing thereby designers from performing real-time simulation.

A new approach is therefore unavoidable to reduce the design process and to enable simulation of complex MEMS structures. In that respect, this paper presents a new approach for modelling linear and especially nonlinear MEMS structures based on Cosserat theory for a better representation of stress in miniaturized systems, especially in the nonlinear regime. The use of Cosserat theory leads to a reduction of the complexity of the modelling and thus increases its capability to simulate microstructures in real-time, indispensable for haptic technology [5]. To demonstrate the feasibility of the proposed model, a cantilever microbeam undergoing loads modelled with ANSYS, SABER and Cosserat theory are compared.

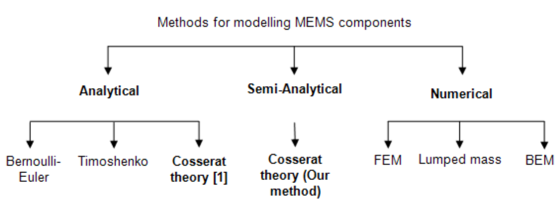

Figure 1: Taxonomy of MEMS modelling methods

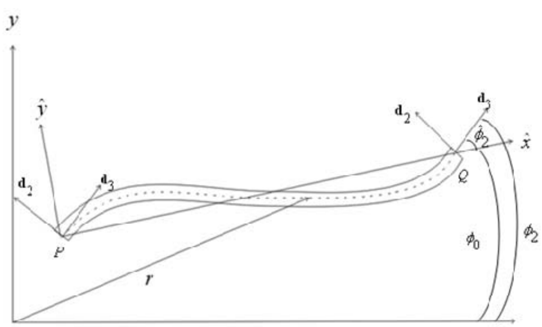

Figure 2: A Cosserat rod: Deformed beam

Figure 2 is a schematic construction of a 2D Cosserat beam element in the x- and y-plane. A Cosserat rod can be described by defining a set of cross-sections the centroids of which are connected by a curve which is referred to as the line of centroids. The motion in space of a nonlinear Cosserat rod segment can be represented as a vector $r(s,t)$, called a Cosserat curve, which describes the position of the line of centroids of the cross-sections (Figure 2, dotted line). The modelling of the microstructures is based on the centroid line and a director replacing a detailed 3D meshing used in FEM. In the Cosserat theory, the accuracy will depend on the method used to model the motion/deformation of the centroid line. Our approach uses a semi-analytical method based on both power series expansions and a multimodal approximate method. Unlike in [1] where the Newton's dynamical law and analytical method are used, our approach is based on a semi-analytical method and on the Euler-Bernoulli equation of motion. To solve the Euler-Bernoulli equation of motion, the displacements u_x and u_y in the transverse and axial directions are expanded in ascending powers of w [2].

$$\mathbf{u} = \begin{bmatrix} u_x \\ u_y \end{bmatrix} = \left(\begin{bmatrix} \mathbf{a}_{0x} \\ \mathbf{a}_{0y} \end{bmatrix} + w \begin{bmatrix} \mathbf{a}_{1x} \\ \mathbf{a}_{1y} \end{bmatrix} + w^2 \begin{bmatrix} \mathbf{a}_{2x} \\ \mathbf{a}_{2y} \end{bmatrix} + w^3 \begin{bmatrix} \mathbf{a}_{3x} \\ \mathbf{a}_{3y} \end{bmatrix} + \ldots \right) \quad (1)$$

The matrices $\mathbf{a}_{ix}$ and $\mathbf{a}_{iy}$ represent axial and transverse displacements, respectively. w is the circular frequency. To validate our design approach, our model is tested in the same conditions as [3]. Table 1 compares the results of the analytical calculations, static analysis of the ANSYS and SABER models [3], with our model. Afterwards, to validate the nonlinear model, we compared the analytical buckling load [4] against the buckling load obtain using our model.

Static Load [μN]	Analytical solution [μm]	ANSYS (100 elements) [3] [μm]	SABER (2 elements) [3] [μm]	SABER (8 elements) [3] [μm]	Our model (2 elements) [μm]	Our model (8 elements) [μm]
$F_y = 7.3*10^{-4}$	2.296	2.298	2.296	2.296	2.296	2.296
$F_x = 7.3*10^{-4}$	$8.971*10^{-7}$	$8.971*10^{-7}$	$8.971*10^{-7}$	$8.971*10^{-7}$	$8.971*10^{-7}$	$8.971*10^{-7}$

a)

	Analytical solution [4]	Our model	Error (%)
1 CRE	$5.42828*10^{-5}$	$5.46911*10^{-5}$	0.7
4 CREs	$5.42828*10^{-5}$	$5.42846*10^{-5}$	0.003
10 CREs	$5.42828*10^{-5}$	$5.42825*10^{-5}$	0.00005

b)

Table 1: a) Static analysis of the linear cantilever microbeam b) Buckling effect

The results show clearly that model is in good agreement with the SABER, ANSYS and analytical solutions (Table 1a). Then the nonlinear model has been used to compute the first buckling load of the microbeam. Table 1b shows that the results obtained for 1, 4 and 10 elements for the discretization of the microbeam are very close to the analytical solution. In this paper, it is demonstrated that the Cosserat theory has been successfully for modelling and testing simple structures.

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