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APPROXIMATION OF THE DISPLACEMENT AND MIXED FORMULATIONS OF SOLIDS WITH EMBEDDED DISCONTINUITIES USING THE MESHLESS ELEMENT FREE GALERKIN METHOD

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ABSTRACT

This paper presents a general variational formulation of the Meshless Element Free Galerkin Method for solids with embedded discontinuities based on the originally proposed by Fraeijs De Veubeke [1] for a continuum solid, *i.e.,*

$$\Pi(u,\sigma,\varepsilon,t)=\int_{\Omega}\left[\boldsymbol{\sigma}:\left(\boldsymbol{\varepsilon}^{u}-\boldsymbol{\varepsilon}\right)+W(\varepsilon)-\mathbf{b}\cdot\mathbf{u}\right]d\Omega-\int_{\Gamma_\sigma}\mathbf{t}^{*}\cdot\mathbf{u}d\Gamma-\int_{\Gamma_u}\mathbf{t}\cdot(\mathbf{u}-\mathbf{u}^{*})d\Gamma \tag{1}$$

and enhanced by Juarez and Ayala [2] for a solid with embedded discontinuities, *i.e.,*

$$\Pi\left(u,\sigma,\varepsilon,t,\left[|\mathbf{u}|\right]\right)=\int_{\Omega\setminus S}\left[\boldsymbol{\sigma}:\left(\boldsymbol{\varepsilon}^{u}-\boldsymbol{\varepsilon}\right)+W(\varepsilon)-\mathbf{b}\cdot\mathbf{u}\right]d\Omega-\int_{\Gamma_\sigma}\mathbf{t}^{*}\cdot\mathbf{u}d\Gamma-\int_{\Gamma_u}\mathbf{t}\cdot(\mathbf{u}-\mathbf{u}^{*})d\Gamma+\int_{S}T\cdot\left[|\mathbf{u}|\right]d\Gamma \tag{2}$$

These formulations have as independent fields: displacements, $\mathbf{u}$, strains, $\boldsymbol{\varepsilon}$, stresses, $\boldsymbol{\sigma}$ and tractions, $\mathbf{t}$. W(ε) is the strain energy density, b is the body force vector and **u*** and t* are, respectively, the prescribed displacements and tractions on boundary Γ. The formulation given in eq. (2) has also as independent variable the displacement jump, $[|\mathbf{u}|]$, on the localization zone ***S***, where the traction, ***T*,** keeps continuity.

In the meshless approximation of the formulation of eq. (2), the actual behaviour of a solid with localised failure zones will not depend on a mesh, as it would be the case of a finite element approximation. This characteristic makes it particularly convenient when applied to problems involving large deformations such as those encountered in damage mechanics where excessive mesh distortions may originate numerical instabilities and a reduction in the accuracy of the approximation in conventional finite element approximations. In this work the meshless displacement and mixed formulations and approximations for a solid with embedded discontinuities are investigated. In the first, where the displacement field is considered as unknown and, in the second, where the displacement and strain fields are considered independent and unknown of the problem. For both, the approximations used for the unknown fields are constructed using a moving least square approach consistent with that used in the Element Free Galerkin

Method for solids without discontinuities [3, 4]. The variational formulations corresponding to the displacement and the mixed displacement-strain approaches are:

$$\Pi\left(u,t,\left[|\mathbf{u}|\right]\right)=\int_{\Omega\setminus S}\left[W\left(\boldsymbol{\varepsilon}^{u}\right)-\mathbf{b}\cdot\mathbf{u}\right]d\Omega-\int_{\Gamma_\sigma}\mathbf{t}^{*}\cdot\mathbf{u}d\Gamma-\int_{\Gamma_u}\mathbf{t}\cdot(\mathbf{u}-\mathbf{u}^{*})d\Gamma+\int_{S}T\cdot\left[|\mathbf{u}|\right]d\Gamma \tag{3}$$

and

$$\Pi\left(u,\varepsilon,t,\left[|\mathbf{u}|\right]\right)=\int_{\Omega\setminus S}\left[\boldsymbol{\sigma}:\varepsilon-W\left(\boldsymbol{\varepsilon}^{u}\right)-\mathbf{b}\cdot\mathbf{u}\right]d\Omega-\int_{\Gamma_\sigma}\mathbf{t}^{*}\cdot\mathbf{u}d\Gamma-\int_{\Gamma_u}\mathbf{t}\cdot(\mathbf{u}-\mathbf{u}^{*})d\Gamma+\int_{S}T\cdot\left[|\mathbf{u}|\right]d\Gamma \tag{4}$$

The independent fields for the above formulations are approximated as

$$\mathbf{u}=\Phi\cdot U+M\left[|\mathbf{u}|\right];\quad \boldsymbol{\varepsilon}=\Psi\cdot\mathrm{E};\quad \mathbf{t}=\tau\cdot\mathrm{T} \tag{5}$$

where U, E and T are the nodal displacements, strains and tractions; and Φ,Ψ,τ the corresponding shape functions defined as in the conventional Meshless Method [3, 4]. M are shape functions for the displacement jump.

To simulate the evolution of damage that a solid with embedded discontinuities undergoes through its evolution to collapse, a continuum discrete damage model is used [5], *i.e.,* traction-displacement jump relation ($T-\left[|\mathbf{u}|\right]$).

It may be observed that the displacement formulation given in eq. (3) leads to a conventional meshless approach where the Lagrange multipliers used to enforce the essential boundary conditions are equivalent to the tractions of this formulation. The advantage of using the displacement approach is its simplicity, even though it has as a disadvantage the fact that when a discontinuity appears an additional tracking formulation is required, somehow overshadowing this simplicity. This disadvantage, however, does not exist in the mixed formulation where an additional formulation for the tracking of the discontinuity is not required even though the number of unknowns is sensibly increased. Regarding the numerical implementation of the investigated meshless element free Galerkin formulations, it has been found shown that this does not present any difficulty as it closely follows what it has already been done by other authors [3,4].

REFERENCES

[1] B.M. Fraeijs de Veubeke, "Diffusion des inconnues hyperstatiques dans les voilures à longeron couples". Bull. Serv. Technique de L'Aéronautique No. 24, Imprimerie Marcel Hayez, Brussels, 56 pp (1951).

[2] G. Juárez and A.G. Ayala, "Variational Formulation of the Material Failure Process in Solids by Embedded Discontinuities Model". To appear, Numer Methods Partial Differential Eq, (2008)

[3] J. Dolbow and T. Belytschko, "An Introduction to Programming the Meshless Element Free Galerkin Method." Archives of Computational Methods in Engineering, Vol. 5, 3, 207-241, (1998).

[4] Y. Chen, J. Lee and A. Eskandarian, *Meshless Methods in Solid Mechanics*. Springer, New York, (2006).

[5] J. Oliver, "On the discrete constitutive models induced by strong discontinuity kinematics and continuum constitutive equations". Int J Solids Struct, Vol. 37, 7207-7229 (2000).