8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

SHAPE-MATERIAL SENSITIVITY FRAMEWORK FOR ELASTIC-WAVE IDENTIFICATION OF PENETRABLE DEFECTS

*** Bojan B. Guzina[1] and Marc Bonnet[2]**

[1] Department of Civil Engineering
University of Minnesota
Minneapolis, MN 55455, USA
guzina@wave.ce.umn.edu
www.ce.umn.edu/∼guzina

[2] Laboratoire de Mécanique des Solides
Ecole Polytechnique
F-91128 Palaiseau Cedex, France
bonnet@lms.polytechnique.fr
www.lms.polytechnique.fr/users/bonnet

Key Words: *Shape-material sensitivity; elastic waves; penetrable obstacles; defect identification.*

ABSTRACT

Non-invasive identification of penetrable solid defects using elastic waves is a topic of considerable interest in mechanics with applications to nondestructive material testing, medical diagnosis, hydrocarbon exploration, and underground object detection. In the context of seismic surveys, comprehensive three-dimensional mapping of subterranean structures is typically associated with the interpretation of a large number (often thousands) of motion measurements using elastodynamic or acoustic models that are inherently based on domain discretization (e.g. Plessix et al., 1999). In contrast, this investigation is concerned with problems where detailed 3D mapping of buried objects is required and only a few surface measurements can be made. In such instances the boundary integral equation (BIE) formulations, which provide a direct mathematical link between the observed waveforms and geometric/material characteristics of a hidden object, can be used to effectively compensate for the limited field data (see Colton and Kress, 1983, for acoustic problems).

The focus of this study is the development of an analytical and computational framework for the identification of penetrable solid defects using an elastodynamic BIE approach. The defects are assumed to be homogeneous and bonded to the matrix; for the purpose of elastic-wave identification, they are characterized by their boundary (i.e. the surface that separates them from the matrix), elastic tensor, and mass density. Following a classical approach, the inverse problem is reduced to the minimization of a cost functional quantifying the misfit between experimental observations (the values of surface displacement at sensor locations) and their simulations for an assumed defect configuration. The predictive model used in this study is based on a coupled system of regularized boundary integral equations (Bonnet, 1999; Pak and Guzina, 1999). For efficiency of the gradient search technique employed by the inverse solution, sensitivities of the cost function with respect to defect parameters are evaluated via an adjoint problem approach which, besides the matter of elegance, offers a superior computational performance relative to finite-difference sensitivity estimates. This is accomplished by generalizing upon the shape sensitivity approach (Guzina et al., 2003; Bonnet, 1995) for impenetrable defects, and introducing an adjoint-field-based *material sensitivity* formula. As the latter is expressed in terms of a domain integral (over the volume of a trial inclusion) and hence is not well suited for applications within a BIE

framework, a *direct differentiation* approach based on the material-parameter derivative of the governing integral equation is also developed for the computation of material sensitivities. A similar BIE-based treatment of the shape and material sensitivities has been recently proposed by Zacharopoulos et al. (2006) for optical tomography, featuring the scalar Helmholtz equation with a complex wavenumber.

For identification purposes, the shape and material sensitivity formulas developed in this study are coupled with a nonlinear optimization algorithm towards the solution of the 3D inverse scattering problem. The numerical results, which employ a direct boundary element method, BFGS quasi-Newton minimization, and the augmented Lagrangian cost functional (as a means to deal with physical inequality constraints), demonstrate the feasibility of identifying the geometry and material characteristics of defects hidden in a semi-infinite solid from a limited number of waveform measurements taken on the material surface. In future applications, the proposed approach may be used as a tool to refine *preliminary* imaging results obtained using non-iterative and computationally fast *sampling* methods for the qualitative identification of penetrable defects (Guzina and Chikichev, 2007; Guzina and Bonnet, 2006; Guzina and Madyarov, 2007).

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