

## REPRESENTATION OF VECTOR-VALUED RANDOM VARIABLES

\*John Red-Horse<sup>1</sup> and Roger Ghanem<sup>2</sup>

<sup>1</sup> Sandia National Labs  
P.O.Box 5800 MS 0828,  
Albuquerque, NM 87185  
jrredho@sandia.gov

<sup>2</sup> University of Southern California  
Los Angeles, CA 90039  
ghanem@usc.edu

**Key Words:** *Stochastic Analysis, Karhunen-Loeve Expansions, Polynomial Chaos Expansions, Stochastic Orthogonal Expansions*

### ABSTRACT

In recent years, orthogonal representations of random variables and processes using the Karhunen-Loeve expansion and the Polynomial Chaos expansions have been used in conjunction of various constrained such as partial differential equations to characterize the probabilistic structure of stochastic processes with particular mathematical structures. In this paper, we present a general description of stochastic processes that naturally permits the integration of a variety of such constraints. This is done by construing the stochastic processes of interest as members of a suitably constructed functional space. Specifically, we extend the Karhunen-Loeve expansion to vector-valued stochastic processes with particular attention to distribution-valued processes. In the process, we explore the structure of the associated reproducing kernel Hilbert spaces (RKHS) and the weak form of the associated eigenvalue problem. We identify the role that the dual of the phase space of the stochastic process plays in the analysis of this problem. We show the significance of our presentations to various model reduction formalisms.