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SPACE-TIME VARIATIONAL (DIS)CONTINUOUS GALERKIN METHOD FOR FREE SURFACE WAVES.

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ABSTRACT

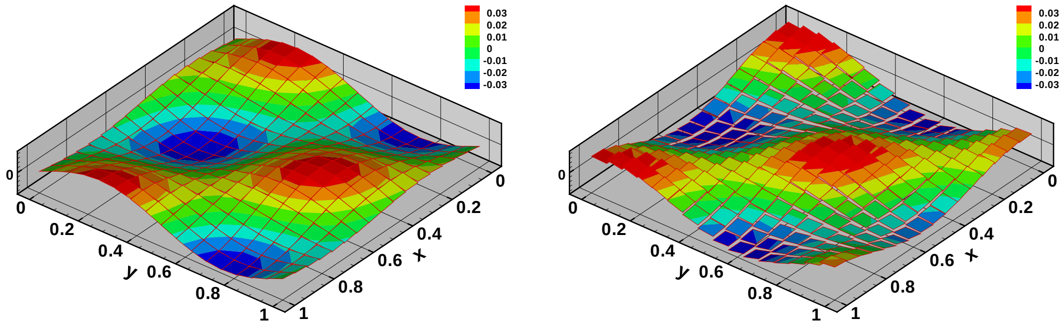
Many water wave problems in marine and offshore engineering are studied using nonlinear free surface wave model which consists of a potential flow equation supplemented with kinematic and dynamic free surface boundary conditions. These equations can be derived from Luke's variational principle [3]. Such a variational principle has the advantage of completely describing the physics of the problem using a single functional and follows the conservation of energy. Therefore, numerical discretization based on the variational principle may conserve discrete energy. Hence, in the present work, we consider a new space-time variational (dis)continuous Galerkin method for free surface gravity water waves.

Space-time discontinuous Galerkin (DG) finite element methods are advantageous for problems arising in fluid mechanics that have moving boundaries or interfaces in the flow domain [1,5]. The main advantage of the method is the flexibility in handling the deforming grids because of the unification of space and time in the numerical formulation. Extra advantages are $hp-$adaptivity and suitability for parallelization. In the present water wave problem, the free surface continuously evolves in time and thus a space-time DG method is useful to handle deforming grids due to the free surface (see [5]).

In the space-time variational DG method, we first establish a primal relation between the approximated velocity field and the velocity potential by following the approach of Arnold et al. [2] or Brezzi et al.[3]. Second, we derive a discrete functional analogous to the continuum functional of the free surface water wave problem using the primal relation. Subsequently, we apply Lukes variational principle on the discrete functional to obtain a discrete variational formulation for the free surface water wave problem. We obtain a consistent variational discretization by approximating the functions to be continuous in time but discontinuous in space. For linear free surface waves, the numerical discretization results into a algebraic system of equations with a compact stencil with a structure similar to that in [5].

The numerical scheme for linear free surface waves is second order accurate for linear polynomial approximations and shows no dissipation qualitatively. In Figure 1, we present our preliminary results

for linear harmonic free surface waves which do not show decay in the maximum amplitude of the free surface waves even when simulated for a long time. We conclude that the present numerical method is stable and accurate which can be extended to the nonlinear free surface waves.



(a) At $t = 0$. (b) At $t = 100T$.

Figure 1: Free surface height of the linear harmonic free surfaces at initial (left) and final time levels. The linear waves are simulated up to 100 time periods with T the time period.

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