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A COMPLEX VARIABLE SYMMETRIC BOUNDARY ELEMENT MODEL FOR POLYGONAL KIRCHHOFF PLATES

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ABSTRACT

Standard boundary element models are usually based on displacement boundary integral equations defined by integrating the differential equations, weighed by fundamental solutions associated with static sources, on the domain. Enforcing the boundary conditions at the collocation points a non symmetric and non definite positive boundary system is obtained. Symmetric boundary element formulations generate symmetric systems that are suitable for coupling with finite elements and analyzing dynamic problems, as they have reliable criteria to check the convergence of the approximation. In these models weighted forms of the boundary integral equations associated with both static and kinematic sources are considered. The entries of the symmetric system are computed by the double boundary integration of the fundamental solutions weighted by the functions approximating the source density distributions taken as being equal to the functions interpolating the boundary variables.

The analytical computation of the coefficients forming the symmetric boundary systems produces accuracy and efficiency in the numerical model, but requires complicated manipulations in cases of generically oriented elements and special techniques for dealing with the singularities of the kernels arising when the integration domains overlap. These problems are particularly serious in the analysis of polygonal Kirchhoff plates owing to the higher orders of singularity and the more complicated expressions of the involved fundamental solutions [1] [2].

The present work aims to develop a procedure for an agile construction of symmetric boundary systems. To this end a complex variable description of the quantities involved in the evaluation of the boundary coefficients proves to be useful for obtaining compact expressions of the integrands and for carrying out in a synthetic way the analytical manipulations of the integration process. A first application of the complex variable description appeared in the work [3] dealing with the symmetric boundary analysis of plane elasticity problems. In the present work this approach is generalized, representing both the

fundamental solutions and the shape functions in the complex plane, introducing suitable integration rules and pointing out the recursive form of the involved integrals. In particular, the present work deals with generic polygonal Kirchhoff plates in order to emphasize the greater compactness of the proposed procedure in comparison with the classical real variable description.

The symmetric boundary element model is constructed by considering the Galerkin weighted forms of the integral equations for the transversal displacement, the normal slope, the bending moment, the equivalent shear and the corner reaction. These equations are written in complex form assuming a complex variable and its conjugate variable for describing the distance between the source and the field points. In this way the double analytical integration of the products between the fundamental solutions and the shape functions can be carried out more concisely than in the real plane. To this end a specific integration rule for complex variables has been developed. This procedure is coupled with repeated transformations by parts on two contiguous boundary elements where the singular boundary terms are canceled exploiting the continuity of the shape functions. The preliminary regularization of the kernels is used for the evaluation of all the boundary coefficients producing a significant reduction in the number of prime integrals.

The use of the complex variable approach improves the performances of the discrete model. In particular, the analytical evaluation of all the system entries for both overlapping and separate integration domains avoids the differences in accuracy arising from a mixed use of analytical and numerical integration. Moreover the boundary system is constructed faster than the real variable procedures.

Some analytical results from the boundary integration of the most singular kernels are reported in order to show the compactness of the analytical results and the advantages of the regularization process. Some numerical results obtained with the computer code implementing this boundary element model allow its accuracy to be tested for analyzing polygonal plates with various boundary conditions.

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