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ROBUST MODEL UPDATING FOR INSUFFICIENT DATA

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ABSTRACT

The increasing need of many industrial fields for highly accurate predictions of performance and reliability gives rise to the need of enhanced underlying mathematical models. Engineers are aware that computational models are only approximations to the reality due to limited physical theory and available data. Hence, to make a decision based on limited, incomplete information poses a challenging problem. Full-scale and even component-wise experimental measurements are very costly and therefore few test samples are usually available. The exorbitant costs even lead some industries, such as the aerospace industry, to limit themselves to one single specimen. Based on information this scarce, reliable conclusions about lower and upper bounds of the response quantities of interest are out of reach.

Recently, a novel approach for coping with insufficient data has been introduced [1], that attempts to extract the maximum amount of information delivered by the available data and processes it using a minimum of additional assumptions. The approach provides information about the confidence of the events of interest based on the available data. In this approach the underlying distribution is established as a function of the number of available data points and an appropriate confidence level providing a safeguard against severe underestimation of the variability of the measured quantities.

The approach has been applied successfully to a static problem [1,2]. This paper shows the extension of the approach to the field of dynamics. Response predictions of dynamical systems are in general very sensitive to the unavoidable scatter of the underlying mathematical model. In order to cope with this challenging problem, a robust stochastic model is constructed such that the sensitivity of the output due to the scatter of the input parameters is minimized. It will be shown how to deal with insufficient information extracted from N measurements by applying kernel densities on modal parameters. The problem of defining correlations between the random modal data has to be addressed when extending the one dimensional kernel density function

$$f_X(x;\sigma) = \frac{1}{\sigma N\sqrt{2\pi}} \sum_{j=1}^{N} \exp\left(\frac{-(x-x^{(j)})^2}{2\sigma^2}\right) \tag{1}$$

to the n-dimensional case of the n modal parameters. The standard deviation can be tuned such that the reliability of the results can be quantified by an apriori defined confidence level α. This yields the

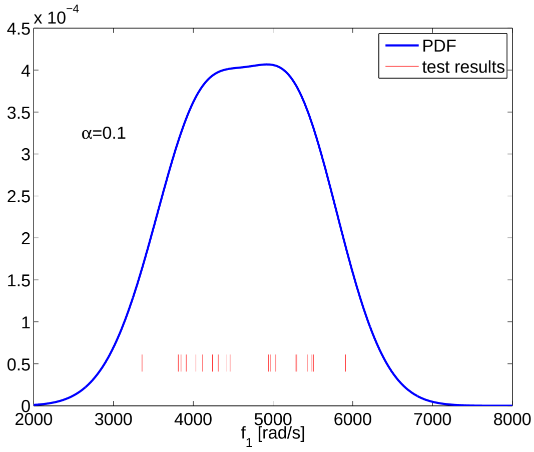

Figure 1: Kernel density function for the first eigenfrequency of the subsystem

equation

$$\int_{-\infty}^{a} f_X(x;\sigma)\,dx + \int_{b}^{+\infty} f_X(x;\sigma)\,dx = 1 - \alpha^{1/N}, \tag{2}$$

where a and b are the lower and upper bounds of the domain covered by the available data. More precisely, they are $a = x_{\min} - (x_{\max} - x_{\min})/(2N-2)$ and $b = x_{\max} + (x_{\max} - x_{\min})/(2N-2)$. In this way, a reliable stochastic modal model for response predictions can be constructed.
As a guideline for the development of the updated mathematical model the recommendations for verification and validation in [3] are used. This document provides the conceptual framework to better assess and enhance the credibility of the constructed model.

As a numerical example the challenge problem for dynamics as proposed in the Sandia National Laboratories Validation Workshop [4] is used. This example encompasses several aspects that arise in real applications and is therefore well suited for the assessment of the suggested approach. More precisely, the structure consists of an elastic founded beam with varying cross-sectional properties and random elastic foundation. A substructure composed by three masses is connected to the beam with a weakly non-linear spring. Exemplarily, the obtained probability density function for the first eigenfrequency of the subsystem is shown in Fig. 1. In order to determine the predictive quality, the obtained model is validated in the subsequent validation process performed on the subsystem and on an accreditation configuration.

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