

Reliability evaluation for uncertain non-linear structural systems under stochastic dynamic excitation.

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Key Words: *uncertain structural system, non-linear, reliability, failure domain, Markov chain Monte Carlo, Linked Importance Sampling*

ABSTRACT

Most structures react to exceptionally high forces caused for example by extreme winds, sea waves, earthquakes, avalanches, etc. in a non-linear way before they finally collapse. These mostly environmental loadings cause dynamic excitations which might be suitably represented by so called stochastic processes. Realistically, not only the the excitation is uncertain, but the parameters specifying the stiffness, damping and sometimes the mass are usually also to some extent uncertain, most of all the parameters characterizing the non-linearity.

The assessment of the reliability of such dynamically excited system with uncertain structural properties is an extremely challenging problem. The difficulties depend, of course, mainly on the complexity of the structural system and degree of non-linearity. The present state of the art is not developed far enough to consider highly non-linear and complex uncertain systems under stochastic excitation which are characterized by high dimensions in the parameter space of the excitation and of the structure. Only stochastic search algorithm such as subset Monte Carlo simulation [1] and variants of it [4], demonstrated the ability of dealing with non-linear uncertain systems subjected to stochastic dynamic excitation [2,3,4]. These black box types of stochastic search procedures are generally applicable and do not exploit any specific mechanical properties of the considered system. Although these procedures are more efficient than direct Monte Carlo, several thousands and more functions evaluations, i.e. dynamic non-linear Finite Element analyses, are usually required for a quantitative reliability estimation.

In the present work, methods based on power inputs of the stochastic excitation will be exploited to identify subsets of the excitation which cause failure. This procedure is based on the simple energy consideration that any excitation which maximizes the mechanical energy of the system have the potential to endanger the integrity of the structure. This method considers the velocity of the displacement field of the structure and the energy dissipation induced by viscous damping, friction and hysteresis. Direct Monte Carlo sampling in the parameter space of the excitation and of the structural properties provides the setting of the initial non-linear dynamic analysis. Starting from the initial excitation, an additional excitation which maximizes the power input into the system will be computed

which drives the non-linear system into failure. Compared with subset simulation which searches randomly (blindly) for the failure domain, the proposed approach exploits the available mechanical laws to find efficiently the domain of failure. Since the complement of the failure domain defines the reliability, this domain needs to be identified. The proposed methodology, is only capable to compute random points in the failure domain which might be helpful in the design process. It does not, however, specify the associated failure probability.

Given independent points in the failure domain, Markov Chain Monte Carlo sampling will be used further to represent the failure domain by more random samples. In a next step, the failure domain will be enveloped by an auxiliary domain, which specifies the safe domain by hyper planes in the parameter space, specified by a point and unit vector of the direction pointing toward the safe domain. Linked Importance sampling [5,6] will be applied to determine the probability of failure for the auxiliary domain. No dynamic Finite Element will be necessary to estimate this probability by Linked Importance Sampling, because only the minimal distance to one of the established hyper planes will be needed. In a final step, the failure probability will be computed by the auxiliary domain method as proposed in [4] employing again Markov Chain Monte Carlo sampling.

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