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REDUCING LARGE UNSYMMETRIC EIGENPROBLEMS IN FLUID-SOLID STRUCTURES BY AUTOMATED MULTI-LEVEL SUBSTRUCTURING

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ABSTRACT

Over the last few years, a new method for huge eigenvalue problems

$$Kx = \lambda Mx,$$

where K and M are sparse symmetric and positive definite matrices, known as *Automated Multi–Level Substructuring (AMLS)*, has been developed by Bennighof and co-authors [1], and has been applied to frequency response analysis of complex structures. Here the large finite element model is recursively divided into very many substructures on several levels based on the sparsity structure of the system matrices. Assuming that the interior degrees of freedom of substructures depend quasistatically on the interface degrees of freedom, and modelling the deviation from quasistatic dependence in terms of a small number of selected substructure eigenmodes, the size of the finite element model is reduced substantially yet yielding satisfactory accuracy over a wide frequency range of interest.

Recent studies in vibro-acoustic analysis of passenger car bodies where huge FE models with more than six million degrees of freedom appear and several hundreds of eigenfrequencies and eigenmodes are needed have shown that AMLS is considerably faster than Lanczos type approaches for this sort of problems [3,5].

On each level of the hierarchical substructuring AMLS consists of two steps. First for every substructure of the current level a congruence transformation is applied to the matrix pencil to decouple in the stiffness matrix the substructure from the degrees of freedom of higher levels. Secondly, the dimension of the problem is reduced by modal truncation of the corresponding diagonal blocks discarding eigenmodes according to eigenfrequencies which exceed a predetermined cut-off frequency. Hence, AMLS is nothing else but a projection method where the large problem under consideration is projected to a search space spanned by a smaller number of eigenmodes of clamped substructures on several levels.

Eigenproblems governing free vibrations of fluid-solid structures have the following form

$$\begin{pmatrix} K_s & C \\ O & K_f \end{pmatrix} \begin{pmatrix} u_s \\ p_f \end{pmatrix} = \lambda \begin{pmatrix} M_s & O \\ -C^T & M_f \end{pmatrix} \begin{pmatrix} u_s \\ p_f \end{pmatrix},$$

where K_s and K_f are the stiffness matrices , and M_s and M_f are the mass matrices of the structure and the fluid, respectively. u_s is the displacement of the structure, p_f is the fluid pressure vector, and C is the coupling matrix between fluid and structure. These unsymmetric eigenproblems are covered in the following way [1,3,5]: one first solves the symmetric eigenproblems $K_s\phi_s = \lambda M_s\phi_s$ and $K_f\psi_f = \lambda M_f\psi_f$ governing free vibrations of the fluid and the structure independently, and the original problem is then projected to the space spanned by some of these eigenmodes. So, the coupling is not considered when constructing the search space, but only in the projected problem.

In this presentation we discuss two variants of AMLS for the fluid-vibration problem: Taking advantage of the fact that the above eigenproblem has real eigenvalues which is inherited by each substructure we incorporate the coupling already into the reduction process [4]. Secondly, the eigenproblem can be transformed to a rational symmetric eigenvalue problem allowing a minmax characterization of its eigenvalues, to which the AMLS approach is applied [2].

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