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An adaptive response surface approach for reliability analyses of high-dimensional problems

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ABSTRACT

In structural design the consideration of uncertainties becomes more and more important. Generally the application of a reliability analysis is very complicated due to the required large number of simulations, where each corresponds to a realization of the random material, geometry or loading properties. For this reason many approximation methods have been developed, which allow a reliability analysis with a smaller number of samples. First order and second order reliability methods (FORM and SORM) are two of this methods, which assume the existence of only one design point and do linear or higher order approximation around the design point. Another well-known method is the response surface method, where the true limit state function is replaced by an approximation function. Early methods have used a global polynomial approximation (e.g. in [1]). Later local approximation schemes as Moving Least Squares, Kriging, radial basis functions and sophisticated global methods as artificial neural networks have been applied. But the generalization of these methods for higher numbers of random variables is still very difficult. Mainly the number of required support points for the approximation increases dramatically with increasing dimension.

The basic idea in utilizing the response surface method is to replace the true limit state function by an approximation, the so called response surface, whose function values can be computed more easily. This requires generally a smooth limit state function. In this work, we want to approximate the indicator function, which has the advantage, that for every case the function values can be determined. Due to the fact, that the indicator function has values, which are only one and zero (failure and safe domain) we only have to classify our samples in two classes. A very efficient tool for classification purposes are Support Vector Machines (SVM), which is a method from the statistical learning theory. The algorithmic principle is to create a hyperplane, which separates the data into two classes by using the maximum margin principle. Approximation methods as interpolating Moving Least Squares [2] and Support Vector Regression are applied if the limit state function can be evaluated everywhere. These approximation and classification methods are suitable in combination with Monte Carlo and other simulation methods. The training data are generated either by stretched Latin Hypercube Sampling, central composite design or fekete sampling, which leads to an almost regular distribution for a given number of training points. Based on the initial setup of training points an adaptive scheme is very promising.

We introduce step-wisely new training points at these samples, which are the points of the classification/approximation function with the shortest distance to the approximated limit state. After each adaptation step the response surface is adapted with the new point. In order to obtain a regular support point distribution along the real classification boundary, we combine this approach with the potential energy corresponding to the existing support points. Regions with small potential energy are far away from existing points. This adaptive method can be applied for smooth and discontinuous limit state functions with single and multiple design points. Very promising for the background sampling scheme is the line sampling approach [3] instead of Monte Carlo or Importance Sampling since the included line search serves directly points on the actually approximated limit state and importance regions are adapted independently. For sets of correlated random variables with non-Gaussian distribution types the approximation scheme is applied in the uncorrelated standard Gaussian space and the set of random variables is transformed by using the Nataf model [4], [5].

In the paper it will be shown that the algorithm converges very fast to an accurate solution of the failure probability, whereby only a small number of samples really have to be calculated. The new samples obtained from the algorithm will be found mainly along the boundary between failure and safe domain. With increasing number of random variables the number of required training data increases only linearly, which enables the investigation of problems with 50, 100 or more random variables with only a moderate number of samples. We will investigate and compare different types of initial designs, different sampling methods for the evaluation of new support points and different approximation methods by means of several numerical examples.

REFERENCES

References

[1] C. Bucher and U. Bourgund. A fast and efficient response surface approach for structural reliability problems. *Structural Safety*, Vol. **7**, 57–66, 1990.

[2] T. Most and C. Bucher. New concepts for Moving Least Squares: an interpolating non-singular weighting function and Weighted Nodal Least Squares. *Engineering Analysis with Boundary Elements*, to appear, 2008.

[3] G.I. Schuëller, H.J. Pradlwarter and P.S. Koutsourelakis. A critical appraisal of reliability estimation procedures for high dimensions. *Probabilistic Engineering Mechanics*, Vol. **19**, 463–474, 2004.

[4] A. Nataf. Détermination des distributions de probabilités dont les marges sont données. *Comptes Rendus de l'Academie des Sciences*, Vol. **225**, 42–43, 1962.

[5] P.-L. Liu and A. Der Kiureghian. Multivariate distribution models with prescribed marginals and covariances. *Probabilistic Engineering Mechanics*, Vol. **1**, 105–112, 1986.