8th. World Congress on Computational Mechanics (WCCM8)
5th. European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 – July 5, 2008
Venice, Italy

LOCAL RADIAL BASIS PARTITION OF UNITY COLLOCATION METHOD

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KEY WORDS: *radial basis function, partition of unity, reproducing kernel function, collocation method*

ABSTRACT

Standard radial basis function [1] offers exponential convergence, however the method is suffered from the large condition numbers due to its ”nonlocal” approximation. The nonlocality of RBF also limits its application to small scale problems. The reproducing kernel approximation [2], on the other hand, provides polynomial reproducibility in a ”local” approximation, and the corresponding discrete system exhibits a relatively small condition number. Nonetheless, reproducing kernel approximation produces only algebraic convergence. This work intends to combine the advantages of radial basis function and reproducing kernel approximation function to yield a local approximation that is more stable than that of RBF, while at the same time offers a higher rate of convergence than that of reproducing kernel approximation. We formulated a localized RBF by introducing a reproducing kernel as the localizing function under the general framework of partition of unity [3]. The error analysis shows that if the error of reproducing kernel is sufficiently small, the proposed method maintains the exponential convergence of RBF, while significantly improving the conditioning of the discrete system and yielding a banded matrix. In two-dimensional Poisson problem we the following condition numbers:

$$\begin{aligned} &\text{Radial Basis Function (RBF)}: && Cond. \approx O(h^{-8}), \\ &\text{Reproducing Kernel (RK)}: && Cond. \approx O(h^{-2}), \\ &\text{Proposed local RBF}: && Cond. \approx O(h^{-3}). \end{aligned}$$

where h is the nodal distance. We see that there exists a significant reduction in condition number in the proposed local RBF compared to the standard RBF. A Poisson problem is given to demonstrate the stability and accuracy of proposed method: $\Delta u(x,y) = (x^2+y^2)e^{xy}$ in $\Omega = (0,1)\times(0,1)$ and $u(x,y) = e^{xy}$ on $\partial\Omega$. In this study, multiquadrics (MQ) RBF, Wendland function $g_{5,3}$ [4] constructed using MQ-RBF, pure RK function with quadratic basis $(p=2)$ and cubic basis $(p=3)$, and the proposed local RBF (L-RBF) constructed by MQ-RBF localized with RK function are

compared. The number of collocation points is selected to be four times of the number of source points (discrete points. The condition number and convergence in L2 error norm are shown in Figures 1 and 2 respectively. Good convergence in the proposed L-RBF is observed. The condition number of the proposed L-RBF is in-between the standard global RBF and the local RK function in coarse discretization. It is observed that the condition numbers increase as discretization is refined in both standard RBF and the Wendland function with large support, whereas the condition numbers in RK and L-RBF are quite insensitive to the resolution of discretization.

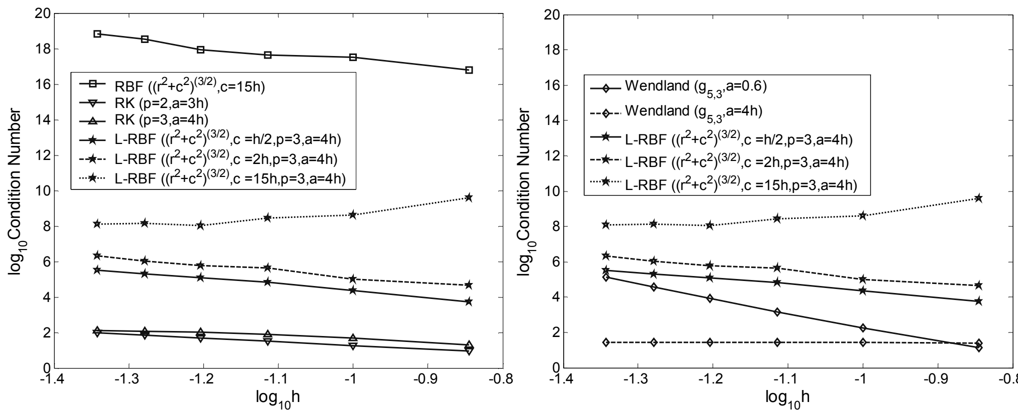

Figure 1. Condition numbers change as refinement in 2D Poisson problem

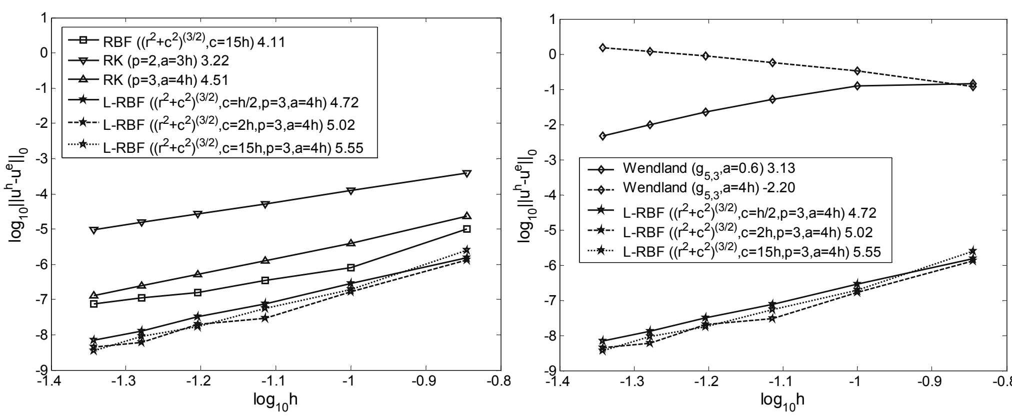

Figure 2. Convergence of L2 error norm in 2D Poisson problem

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