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Scalable Domain Decomposition Algorithms for Coupled Multiphysics Problems

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ABSTRACT

Scalability is one of the most important issues in parallel computation when the size of the problem is large and when the number of processors is large. Domain decomposition methods are very useful for the partitioning of a large problem into many independent subproblems and for solving the problems on large scale parallel computers. The scalability of the methods are well-studied for scalar elliptic equations [5]. In this work we investigate these methods for solving some much harder, coupled nonlinear system of equations arising from the discretization of multiphysics problems.

Many nature and physical phenomena can be simulated on computers by solving partial differential equations which describe the interplay of the physical variables such as pressure, velocity, energy, etc. The physical variables are often coupled in the sense that if one of them changes at a given time and a point in space other variables change at the same time. However, due to the limitation of computing resources, the physical variables are often simulated separately using techniques such as operator splitting or separation of variables. Using such splitting techniques, each field variable is solved individually. Subiterations are required between the subsystems. The subsystems are easier to solve than the global coupled system, but the iterations between subsystems are sequential. The focus of this work is to investigate a fully coupled approach without splitting the system into subsystems. Such an approach is more parallel than the splitting method, but imposes a lot more pressure on the linear and nonlinear solution methods. We will show that with a powerful domain decomposition based preconditioner the convergence of the iterative methods can be obtained even for some difficult cases.

We will report the performance of the algorithms for solving several classes of problems including some simulation problems such as incompressible Navier-Stokes equations, fluid-structure interaction problems, magnetohydrodynamic problems, and also some optimization problems with PDE constrains such as inverse elliptic equations and the control of the incompressible flow problems. Some of the test cases can be found in the recent papers [1-4].

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