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ERROR ESTIMATES FOR ELLIPTIC CONTROL PROBLEMS WITH FINITELY MANY CONSTRAINTS

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ABSTRACT

We consider a class of optimal control problems for semilinear elliptic equations with homogeneous Dirichlet boundary conditions in polygonal 2-dimensional domains, where the controls are taken from a finite-dimensional admissible set and state constraints are required in finitely many points of the associated domain. The state equation of the problem is approximated by a standard finite element method with piecewise linear and continuous ansatz functions. Our main goal is to estimate the error of the approximated control, i.e. the distance of the optimal control of the approximated control problem to the one of the original problem.

Since the controls can be identified with vectors of a finite-dimensional Euclidean space and the space of constraints is finite-dimensional, too, the error is expected to exhibit the same order as the L^∞-error for the finite element scheme. To show this, we extend an optimal L^∞-estimate of the order $h^2|\log h|$ obtained for linear equations by Rannacher and Vexler [2] to the semilinear case. To do so, we also rely on an L^∞-estimate of the order h derived by Casas and Mateos [1]. Next, we apply a perturbation analysis for finite-dimensional optimization problems that is based on generalized equations and the Robinson implicit function theorem. In this way, we are able to show an error estimate of the order $h^2|\log h|$ for the optimal control. The result is confirmed by numerical examples.

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