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Numerical modelling of geophysical fluid dynamics with adapting unstructured meshes

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ABSTRACT

Recently there has been growing interest in the use of unstructured mesh based methods for geophysical fluids applications, in particular ocean modelling [2]. Moving from the structured meshes, typically employed by most models, to unstructured meshes offers many potential benefits. In particular it allows for an excellent representation of complex coastlines and bathymetry [1], and the ability to use wildly different resolutions in different parts of the domain. For example enhanced resolution may be employed to better resolve important localised phenomena such as boundary layers (below) and overflows, and also regions of particular socio-economic or scientific importance [4]. Importantly, unstructured meshes allow for the efficient representation of dynamical interactions between a range of coupled spatial scales. Due to their geometric flexibility this can be achieved with unstructured meshes without resorting to the grid nesting due to the fact that smooth variations in resolution are easily achieved.

Unstructured meshes are also the natural framework within which to formulate adaptive mesh capabilities [3]. Dynamically adaptive methods can be used to optimally resolve and track the formation and evolution of localised features in a priori unknown and/or evolving locations. This would be impossible with any fixed mesh, whether unstructured or not. When utilising adaptive algorithms a model is able to automatically allocate computational resources in an optimal and dynamic manner, as dictated by evolving solution fields and estimates of model and discretisation errors. The aim is that this will lead to more efficient calculations, i.e. overall less degrees of freedom would be required to yield a particular solution to a given accuracy; also for a given computational resource a hope is that it should allow problems to be solved which are more complex than is currently feasible.

In this presentation we will describe some of our experiences with constructing a three-dimensional nonhydrostatic finite element ocean model using fully unstructured adaptive and moving mesh techniques and its application to a range of problems. Of particular importance and focus here are mesh generation; mesh optimisation operations and mesh anisotropy; and error measures.

With any new modelling approach validation is crucial. One simple validation case presented here involves the steady state wind driven barotropic circulation in a rectangular domain. Stommel [6] was the first to describe why one sees the intensification of boundary currents on the western side of ocean basins. Following the notation of [5] consider the streamfunction equation in the domain $[0,1]^2$,

$$\nabla^2\Psi + \alpha\frac{\partial\Psi}{\partial x} = \gamma\sin(\pi y), \quad \alpha = \frac{\beta}{R}, \quad \gamma = \frac{F\pi}{R},$$

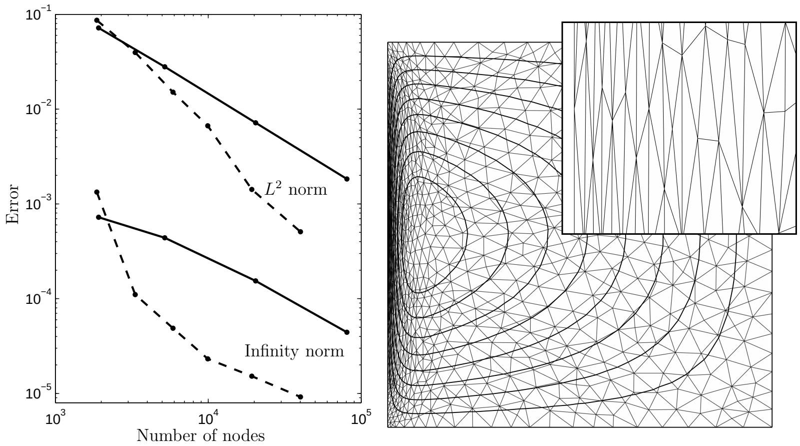

Figure 1: Error between analytical and numerical solutions to Stommel's problem with uniform refinement (solid lines) and anisotropic adaptive refinement (dashed line). The figure on the right shows an example adapted mesh, contours of the computed streamfunction, and a blow up of the mesh near the western boundary where anisotropic refinement may be seen.

with homogeneous Dirichlet boundary conditions. Here $\beta = 50$ is the North-South derivative of the assumed linear Coriolis parameter, $F = 0.1$ is the strength of the wind forcing which takes the form $\tau = -F\cos(\pi y)$, and $R = 1$ is the strength of the assumed linear frictional force. The analytical solution to Stommel's streamfunction equation in the above form is given by

$$\Psi(x,y) = \gamma\left(\frac{1}{\pi}\right)^2 \sin\left(\pi y\right)\left(pe^{Ax} + qe^{Bx} - 1\right),$$

$$p = \frac{1-e^B}{e^A - e^B}, \quad A = -\frac{\alpha}{2} + \sqrt{\frac{\alpha^2}{4} + \pi^2}, \quad B = -\frac{\alpha}{2} - \sqrt{\frac{\alpha^2}{4} + \pi^2}.$$

Figure 1 shows a comparison of results obtained with uniform and anisotropic adaptive refinement. The form of the streamfunction can be seen to yield a velocity field with strong shear in the direction normal to the boundary. The error measure and mesh optimisation algorithm used here yield a mesh which has long thin elements aligned with the boundary. The plot of errors shows the improved results obtained with the use of anisotropic mesh adaptivity compared to uniform resolution.

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