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ANALYSIS OF PIEZOELECTRIC SOLIDS WITH AN EFFICIENT NODE-BASED SMOOTHING ELEMENT

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ABSTRACT

Piezoelectric materials have a wide range of modern engineering applications such as mechatronics, smart structures and microelectronics technology. Thanks to their ability to convert electrical to mechanical energy and vice versa, they are extensively utilized as sensors (passive) or actuators (active) or both at different times to monitor and control vibration, noise and shape of structural systems.

Great progress has been made over past decades towards better understanding of electromechanical coupling behaviour of piezoelectric materials including analytic/numerical methods and experimental models. Although the finite element method (FEM) is considers as a versatile and effective numerical method, there often exists difficulties and bad deteriorations when mesh distortion occurs. On the other hand, several mesh-free methods have become an alternative approach. A recent technique is the stabilized conforming nodal integration (SCNI) mesh-free method [1]. The application of the SCNI in the FEM was presented by Liu *et al.* [2] for 2D elasticity and further for piezoelectric problems by Nguyen-Van *et al.* [3]. It is found that the FEM, integrated with the SCNI technique, archives more accurate results as compared with the conventional one without increasing the modelling and computational costs.

In this paper, we propose a novel approach to incorporate the SCNI technique into the FEM to formulate an efficient node-based smoothing element for analysis of planar electromechanics. The proposed elements are generated from a given general FE mesh of triangular or quadrilateral elements and associated with a single node as shown in Figure 1. The generated elements are similar to the uniform strain approach proposed by Dohrmann *et al.* [4]. The displacement fields of the element are approximated by linear interpolation functions of the original mesh while the approximations of mechanical strains and electric potential fields are normalized using SCNI technique over each smoothing cell (smoothing element) associated with a single node. In particular, the strain ($\tilde{\boldsymbol{\epsilon}}^k$) and the electric ($\tilde{\mathbf{E}}^k$) field used to computed the stiffness matrix are computed by a weighted average of the standard strain and electric field of the FEM as follows.

$$\tilde{\boldsymbol{\epsilon}}^k(\mathbf{x}^k) = \frac{1}{A^k}\int_{\Omega^k} \nabla_s \mathbf{u}(\mathbf{x}) d\Omega = \frac{1}{A^k}\int_{\Gamma^k} \mathbf{n}_u^k \mathbf{u}(\mathbf{x}) d\Gamma,\ \tilde{\mathbf{E}}^k(\mathbf{x}^k) = -\frac{1}{A^k}\int_{\Omega^k} \nabla \boldsymbol{\phi}(\mathbf{x}) d\Omega = -\frac{1}{A^k}\int_{\Gamma^k} \mathbf{n}_\phi^k \boldsymbol{\phi}(\mathbf{x}) d\Gamma,$$

where $\mathbf{n}_u^k$ and $\mathbf{n}_\phi^k$ are outward normal matrices on the boundary Γ^k of Ω^k, $\mathbf{u}(\mathbf{x})$ and $\boldsymbol{\phi}(\mathbf{x})$ are approximated linear functions of the FEM, and $A^k = \int_{\Omega^k} d\Omega$ is the area of a smoothing cell. This method allows field gradients to be directly computed from interpolated shape functions by using boundary integrations along each edge of the smoothing element. No mapping or coordinate transformation and derivatives of shape functions are necessary so that the original meshes can be used in arbitrary shapes. Moreover, the present elements do not introduce any additional degrees of freedom and stresses can be evaluated at field nodes.

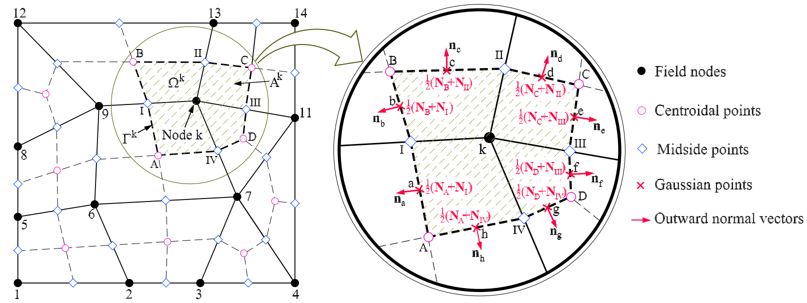

Figure 1: Node-based elements: Subdivision of a typical mesh into smoothing elements associated with nodes. The dashed lines are formed by connecting midside points with centroidal points and serve as new cell (element) boundaries.

Several numerical examples and comparative studies with analytic solutions are investigated to demonstrate the capability, efficiency and reliability of the novel elements. One of them is shown in Figure 2– 6. It is found that the proposed element is efficient and uniformly accurate in assessing static behaviours of 2D electro-mechanics even with extremely distorted meshes.

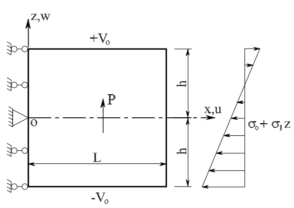

Figure 2: A piezo-strip subjected to a linear stress and an applied voltage [3].

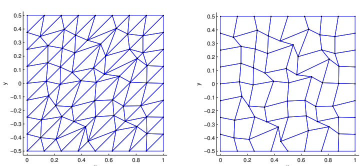

Figure 3: Typical meshes: triangular mesh and quadrilateral mesh.

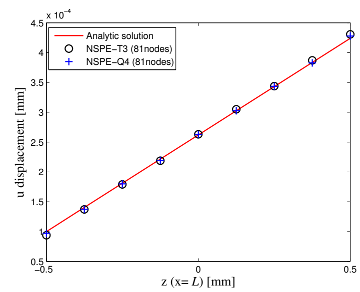

Figure 4: Computed and exact u− displacement.

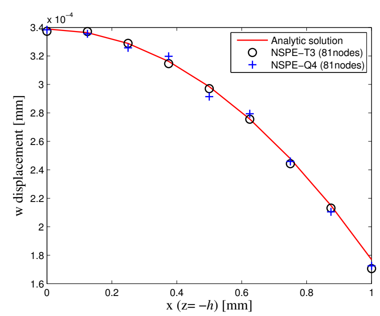

Figure 5: Computed and exact w− displacement.

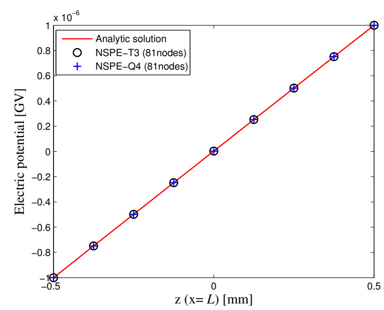

Figure 6: Computed and exact electric potential.

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