8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

Fundamental and applicative challenges in the modeling and computations of shells: an overview

Dominique Chapelle

INRIA, Macs project-team
Rocquencourt, B.P. 105, 78153 Le Chesnay cedex, France
Dominique.Chapelle@inria.fr

Key Words: *Shells, Finite Elements, Locking, Asymptotic Behavior, General Shell Elements*

ABSTRACT

As is well-known in engineering practice, shell structures may produce dramatically different responses – especially when the shell thickness is rather small compared to other characteristic dimensions – depending on their geometries and boundary conditions, in particular. The key to understanding these phenomena is to analyse their *asymptotic behaviors*, namely by considering a sequence of problems indexed by the thickness parameter that we vary while maintaining the midsurface geometry and the boundary conditions fixed. We then find that shell structures undergo two main types of very distinct asymptotic behaviors, namely, bending-dominated or membrane-dominated, see [1] and references therein.

Of course, when performing finite element analyses of shell structures [2], we would like the discrete solutions to accurately reflect the diversity of the above behaviors. More precisely, since we only discretize the problem over the midsurface (i.e. not across the thickness), we expect an accuracy that would only depend on criteria prevailing in 2D analysis, namely, uniform convergence in the surface discretization regardless of the thickness parameter.

However, it was soon recognized in the development of structural analysis procedures that standard finite element techniques – such as displacement-based shell finite elements – fail to display uniformly converging behaviors in general, and that instead finite element approximations tend to dramatically deteriorate when the thickness of the structure decreases [3,4]. In fact, when pursuing the above objectives one faces a dilemma which we now outline. When considering a bending-dominated structure, the numerical difficulty to deal with is *numerical locking*, since the asymptotic behavior then corresponds to a penalized formulation such as for nearly-incompressible elasticity. In order to treat locking, one is led to resorting to *mixed formulations*, which amount to relaxing the penalized constraints by modifying the numerical energy [5]. However, this modification induces a consistency error that is of major concern in membrane-dominated behaviors. Some quadrilateral shell elements representing a satisfactory compromise in this dilemma have been proposed: the MITC ('Mixed Interpolation of Tensorial Components') family [6,7]. Unfortunately, the formulation of effective triangular shell elements is still more difficult, whereas triangular elements are much needed in practical applications where complex geometries must be handled [8].

In this general perspective, we will also present some results and challenges directly related to applicative concerns and numerical practice, in particular as regards general shell elements and 3D-shell elements. These are finite element procedures formulated using 3D constitutive equations and variational principles, which makes them more versatile and easier to implement in engineering practice than methods directly based on shell models – although some underlying mathematical shell models have also been identified for these 3D-based procedures [9,10].

REFERENCES

[1] D. Chapelle and K.J. Bathe. *The Finite Element Analysis of Shells – Fundamentals*, Springer-Verlag, 2003.

[2] M. Bernadou. *Finite Element Methods for Thin Shell Problems*, John Wiley & Sons, 1996.

[3] K.J. Bathe. *Finite Element Procedures*, Prentice Hall, 1996.

[4] T.J.R. Hughes. *The Finite Element Method : Linear Static and Dynamic Finite Element Analysis*, Prentice Hall, 1987.

[5] F. Brezzi and M. Fortin. *Mixed and Hybrid Finite Element Methods*, Springer-Verlag, New York, 1991.

[6] K.J. Bathe and E.N. Dvorkin. "A formulation of general shell elements—the use of mixed interpolation of tensorial components". *Internat. J. Numer. Methods Engrg.*, 22:697–722, 1986.

[7] M.L. Bucalem and K.J. Bathe. "Higher-order MITC general shell elements". *Internat. J. Numer. Methods Engrg.*, 36:3729–3754, 1993.

[8] L. Beirão da Veiga, D. Chapelle, and I. Paris Suarez. "Towards improving the MITC6 triangular shell element". *Computers & Structures*, 85:1589–1610, 2007.

[9] D. Chapelle and K.J. Bathe. "The mathematical shell model underlying general shell elements". *Internat. J. Numer. Methods Engrg.*, 48(2):289–313, 2000.

[10] D. Chapelle, A. Ferent, and K.J. Bathe. "3D-shell finite elements and their underlying model". *M3AS*, 14:105–142, 2004.