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Black-Box Multigrid Preconditioning for Unsteady Incompressible Flows

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ABSTRACT

Simulation of the motion of an incompressible fluid remains an important but very challenging problem. The resources required for accurate three-dimensional simulation of practical flows test even the most advanced computer hardware. The necessity for reliable and efficient solvers is widely recognised. This talk will focus on two components of such a solver: the error control used for self-adaptive time stepping; and the linear solver used at each time level.

Conventional codes typically use semi-implicit time integration leading to a Poisson or Stokes-type system at every time step, but with a stability restriction on the time step. Our alternative approach is a stable version of the TR–AB2 "smart integrator" originally developed by Gresho in the 1980's. Such fully-implicit time integration methods have no restriction on the time step, but have only become feasible in the last five years because of developments in solution techniques for the linear (or linearized) Oseen systems that arise at each time level. To this end, the preconditioning framework that we propose offers the possibility of uniformly fast convergence independent of the problem parameters (namely; the mesh size, the time step and the Reynolds number). In contrast, conventional multigrid solvers for this problem tend to work well if the time step is sufficiently small, but efficiency deteriorates rapidly if either the time step or the Reynolds number is increased.

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