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Tridiagonal factorization algorithm for Chebyshev-tau method with an exponential coordinate mapping

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ABSTRACT

Spectral numerical methods, which involve seeking the solution in terms of a series of known, smooth functions such as, for example, Chebyshev polynomials, are well known in Computational Fluid Mechanics for their superior accuracy [1]. However, the practical use of spectral methods depends critically on the efficiency of algorithms for solution of the resulting systems of algebraic equations. Spectral methods often yield systems of linear equations with dense matrices whose solution may be considerably less cost efficient in comparison to the band matrices resulting from finite element or finite difference techniques. Exception is the Chebyshev-tau method which combined with the recurrence relations for derivatives of Chebyshev polynomials results in tri- and penta-diagonal matrices for approximation of second derivative operators in Cartesian and polar coordinates, respectively [1].

In the present study we derive a similar reduction to a tridiagonal matrix for a Chebyshev-tau approximation of the second derivative operator in a transformed coordinate $\xi = 1 - 2e^{-\alpha x}$ mapping a semi-infinite domain $x \in [0, \infty]$ onto $\xi \in [-1, 1]$ with parameter $\alpha > 0$, which is useful for the numerical solution of boundary layer equations. This approach combined with FFT results in a cost-efficient and accurate algorithm for a direct numerical simulation of boundary layer flows. Application of this algorithm is demonstrated for a turbulent Hartmann-layer flow of an electrically conducting liquid in a transverse magnetic field.

REFERENCES

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