# THE DYNAMICS OF SHOCK AMPLIFICATION THROUGH MULTIPLE IMPACTS

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## ABSTRACT

This paper examines the dynamics of velocity amplification through pair-wise collisions between multiple masses in a chain, in order to develop guidelines for the design of useful machines. For instance, shock-testing machines based on this principle could be extremely versatile: low-cost; providing a very large range of shocks for testing; and a safer and more precise alternative to ballistic testing for the very-high acceleration shock-testing of inherently rugged emerging electronic devices like MEMS, nano-optics devices, and sensors.

A theoretical basis for determining the number and mass of intermediate stages in such a velocity amplifier, modeled as a horizontal chain or a vertical stack of monotonically decreasing masses arranged such that impacts between masses are pairwise, is proposed. The analysis (presented in detail in [1]) is based on simple but adequate rigid body mechanics (that which governs momentum transfer and energy dissipation during impacts between two point masses or rigid spheres) and similar in spirit to that developed in [2,3,4,5]. The influence of mass ratios and the coefficient of restitution on the optimization of the system is identified and investigated. Two cases are examined in particular:

(1) The velocity of the final mass in the chain (that would have the object under test mounted on it) is maximized by defining the ratio of adjacent masses according to a power law relationship. It is shown that such a configuration leads to maximum velocity gain (MVG)..

(2) The energy transfer efficiency of the system is maximised by choosing the mass

ratios such that all masses except the final mass come to rest following impact. It is shown that such a configuration leads to maximum energy transfer (MET).



### Figure 1: 3D plot of ratio (MVG/MET) of velocity gains versus coefficient of restitution and mass ratio. The velocity gain for the MVG system is that of the optimal MVG system for the given R, e.

Comparisons are drawn for various performance and design factors between the MVG and MET configurations as shown in figures 1, 2 and 3. The results are used in proposing design guidelines for optimal shock amplifiers. Although designing an optimal shock-amplifier requires evaluating both MVG and MET configurations, it is shown that for most practical systems, a shock amplifier with mass ratios based on a power law relationship is optimal and can easily yield velocity amplifications of a factor 5-8 times. A prototype shock testing machine that was made using the above principles is briefly demonstrated.



Figure 2: 3D plot of ratio (MVG/MET) of kinetic energy transfer efficiencies for the topmost mass versus coefficient of restitution and mass ratio. The kinetic energy transfer efficiency for the MVG system is that of the optimal MVG system for the given R, e.



Figure 3: 3D plot of ratio (MVG/MET) of total system masses versus coefficient of restitution and mass ratio. The total mass for the MVG system is that of the optimal MVG system for the given R,e

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