# Modeling large networks of fibers

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An important problem in multiscale analysis is to develop continuum (homogenized) models of fiber-matrix composites. In a few special cases, this is possible using purely analytic techniques. For most homogenization methods, however, it is necessary to carry out modeling and simulation of an explicit microscale model in order to determine material parameters or other aspects of the homogenized model.

This motivates consideration of the problem of modeling a composite material composed of a matrix with a large number of embedded fibers. We assume the composite is isotropic.

Our computational model consists of a finite element model of the matrix and a discrete network model of the fibers. The discrete model and continuum model use the same set of nodes, and the stiffnesses of the model are essentially summed in the model.

We discuss several issues that arise in one-dimensional models of fibers including compression versus tension and correct characterization of bending stiffness.

In order to generate a particular geometric instance of this model, one possible approach is to insert fibers at random positions in the matrix and then triangulate the resulting geometric object. For such a geometry, however, most mesh generators would require huge numbers of tetrahedral elements.

This motivates a second approach in which the three-dimensional mesh of the matrix is generated first, and then some edges of the tetrahedra are selected to be fibers. With this approach, it is possible to carry out simulations involving millions of fibers in a uniprocessor environment.

An interesting issue arises concerning isotropy of the fibers. For some common meshes in three dimensions, the edges have only a small number of distinct orientations and hence would introduce an unphysical amount of

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anistropy in the model. We propose the use of a tetrahedral mesh based on the *quaquaversal tiling* [1], which asymptotically includes edges of all orientations in the mesh.

We report on computational experiments for determining electrical conductivity (i.e., solving Laplace's equation) and stiffness in such models. We will also discuss extensions of the model to cover damage.

## References

[1] J. H. Conway and C. Radin. Quaquaversal tilings and rotations. *Inventiones Mathematicae*, 132(1):179–188, 1998.