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Mechanical properties of cellular structures formed by the Voronoi-partition of randomly disturbed cubic lattices.

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ABSTRACT

Interaction between foam or sponge-like materials and classical solids has important applications in fields ranging from orthopaedics to airspace engineering [1,2]. To analyse such interactions in a fairly general way, micromechanical models for the behaviour of the porous materials are needed, which, in order to be broadly applicable, must rely on some simplification of the actual spatial distribution of solid which is affected by systematic gradients as well as statistical variation in true physical systems. Such considerations have lead the authors to explore the concept of a "statistically equivalent microstructure" for the case of cancellous bone, with the goal of retaining both systematic and statistical variations in microstructure while greatly simplifying the calculations and retaining the possibility to apply the method to the large range of microstructural configurations found in the human skeleton.

The paper demonstrates the feasibility of the proposed method by analysing a series of models consisting three-dimensional structures constructed from cylindrical beams by means of commercial finite-element software. The structures are obtained by seeding points along a simple cubic, FCC or BCC lattice and applying a random displacement with Gaussian distribution to each point. The volume is then partitioned into Voronoi-cells [3], the edges of which are converted into one-dimensional beam elements and cut into a cube shape. This cube is subject to a uniaxial compression or simple shear simulation, which allows determining the elastic properties of the virtual structure. A total of 27 models were analysed, using three values for the standard deviation of the Gaussian disturbance for each seeding lattice. An example of the deformed structure, seeded on a FCC-lattice and disturbed by a distribution whose standard deviation equals 0.1 times the unit cell is shown in fig. 1.

The individual result shown in Fig. 1 can be useful in the prediction of damage in the lattice, as stresses in each beam can be determined an criteria for breaking or beam collapse can be applied. However, at the present stage of the research, the authors are more interested in the average values for the elastic constants of the complete structure as a function of model parameters [4,5]. Young's modulus is highest for FCC-based lattices and lowest for simple cube. Its value invariably decreases with increasing

standard deviation; its dependence on the magnitude of the disturbance is strong for the simple cubic seeding lattices and minimal for the FCC case. Poisson's coefficient was found to be relatively independent from the structural details.

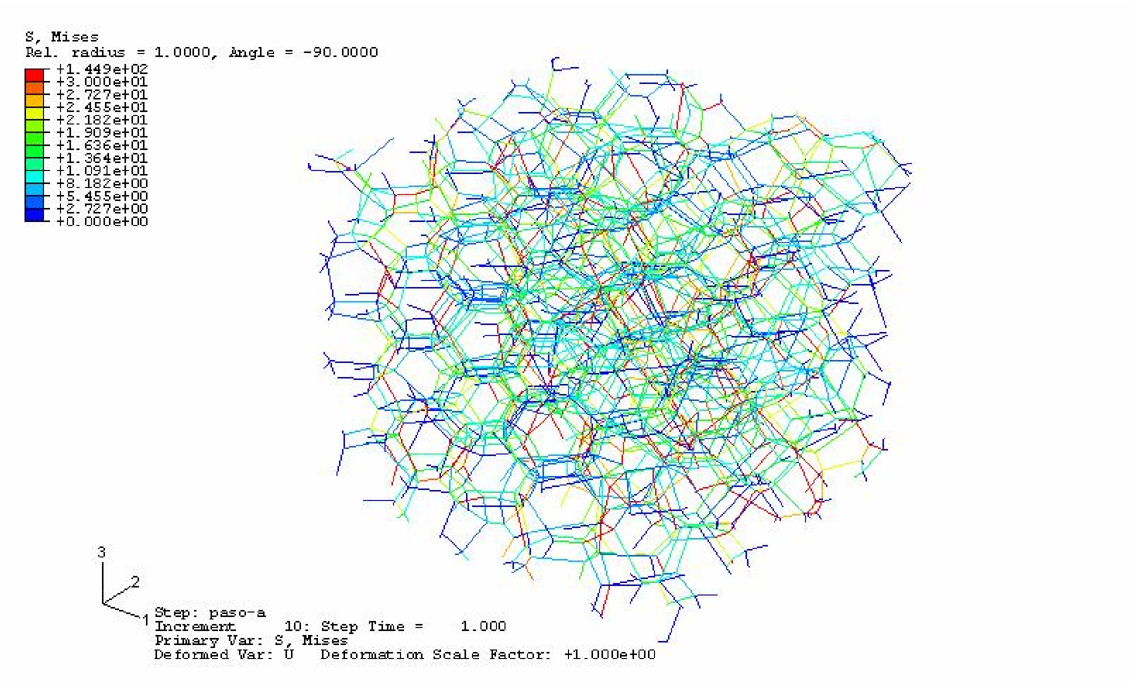

Fig. 1. Virtual sample obtained from the edges of the Voronoi-partition of an FCC seeding lattice disturbed by random vectors following a Gaussian distribution with standard deviation equal to one tenth of the unit cell parameter.

Further steps in the development of this method are described briefly. Extensions to non-cubic seeding lattices consist of simple algebraic manipulations. Conversion to a Monte-Carlo type simulation only requires some relatively simple programming efforts. Introduction of plate-like components in addition to beams, as seen in cancellous bone, is straightforward. At this moment, the main challenge seems to be the determination of a sufficient set of stereological data to guarantee that the virtual structure becomes a true, statistically equivalent representation of the physical system.

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