## MODELING OF DISCONTINOUS FAILURES IN BEAMS AND PLATES

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## ABSTRACT

The ever increasing demand to build economically acceptable structures pushes design of structural systems to their limit. In this situation, the need to better understand the behavior of complex structural systems, including the failure modes, is enormous. This naturally leads to development of new models, which are capable of describing the localized effects, since the failure of a structure, or its components, usually occurs due to the localization of the yielding and damage.

Our work on finite element modeling of discontinous failures in ductile beams and plates is closely related to the embedded discontinouity finite element method, described e.g. in [5] and [6], and specialized for beams e.g. in [4], and for plates e.g. in [3]. To model localized failure of ductile (metal) beams and plates, both diffused plasticity mechanism, e.g. [1], [2], which describes the first part of material nonlinear behavior, and the localized plasticity mechanism, which captures the softening phase, are taken into account. Diffused plasticity is defined at the level of beam/plate stress resultants with state variables describing general isotropic hardening. Localized effects are captured in the form of softening plastic hinges or softening plastic lines. The plastic hinges and plate lines are defined as strong discontinuities of the generalized displacements at the element level. They are treated as additional unknowns that can be eliminated from the global solution scheme through the static condensation procedure. Kinematic parameters, describing localization effects, are correlated with stress resultants acting at the discontinouity by localized softening law.

For an illustration, we present an example of localized failure of a cantilever plate. A plate with length 10, width 5 and thickness 1 is clamped along one of the shorter sides. Transversal displacement is incrementally applied at the opposite end. Plate's response is initially linear elastic with Young's modulus 1 and Poisson's ratio 0.3, until the discontinouity criterion is violated and the softening line forms. The response of the hinge line is determined with yield function  $\phi_{\Gamma} = |m_{crack}| - (m_u - q_s) \le 0$ , where  $m_{crack} = \mathbf{t} \cdot \mathbf{n}$ ,  $\mathbf{t}$  is stress-resultant (moment) traction at discontinuity,  $\mathbf{n}$  is the unit normal vector (determined with the direction of principal bending moment) to the hinge line,  $m_u = 0.00025$  is the ultimate bending strength and  $q_s$  is variable related to

softening. We assume linear softening law with softening modulus  $K_s = 0.008$ . In Figure 2 the reaction-displacement curves for several meshes are plotted. Figure 3 presents hinge line patterns for different meshes.



Figure 2: Reaction-displacement curves



Figure 3: Hinge lines for different meshes (only the area near support is plotted)

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