## MODELLING THE INTERFACE CONDITIONS IN LIMIT ANALYSIS APPLICATIONS

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## ABSTRACT

In many stability problems it is important to account for interface conditions between two or more adjoining bodies, e.g. retaining walls and footings with no-tension contact between soil and structure. These interfaces can be considered as discontinuities in stress and velocity fields developed in the system undergoing plastic collapse. Discontinuous variable fields are routinely employed in FE lower and upper bound limit analyses to improve the performance of lower order elements used to obtain rigorous bounds on the collapse factor. Traditionally, stress and velocity discontinuities have been implemented as a set of special equalities on the stress and velocity variables of adjacent nodes across inter-element boundaries. The major drawback of this approach is that the velocity discontinuities are restricted only to materials with Tresca or Mohr-Coulomb yield criteria. Recently, however, Krabbenhøft et al. 2005 have shown that velocity discontinuities can be represented by a patch of regular elements of zero thickness. This development opens the way for a discontinuous upper bound formulation to be used with general yield criteria in both two- and three-dimensions. By also treating stress discontinuities as a patch of zero thickness elements in a lower bound formulation (Lyamin et al. 2005), lower and upper bound FE methods can be used effectively to solve stability problems involving a wide variety of materials and interface conditions.

The arrangement of the elements at an interface and the locations of the stress and velocity nodes are presented in Figure 1. Assuming an associated flow rule, the interface conditions are governed by the yield function in the zero thickness elements. For upper bound analysis, only one layer of discontinuity elements in the interface is needed as



Figure 1. FE mesh arrangement for upper (a) and lower (b), (c) bound formulations.

these elements have separate stress variables (Figure 1a). For lower bound meshes this is not the case and two options are possible. The first option is illustrated in Figure 1b, where two layers of zero thickness elements are needed to prescribe material properties to the stress points which are separate from the stress points of the domains adjoining the interface. The second variant is to add the interface yield function to the set of yield functions for some of the adjacent boundary stress points (Figure 1c). This yield function must be applied to the interface plane stresses only. Just one layer of zero thickness elements is needed with this approach, making it convenient for applications. To demonstrate the feasibility of the approach two limit analysis problems in soil mechanics and metal forming are considered. The first problem is the plane strain collapse of a surface footing on clay subject to vertical eccentric loading (Figure 2). Two kinds of interface conditions are modelled: full adhesion and tension cut off. The collapse mechanisms shown and corresponding bearing capacity values demonstrate the influence of the cut-off condition.



Figure 2. Strip footing on clay subject to eccentric (e=0.6) loading with full adhesion and tension cut-off interface conditions.

The second example is that of equal-channel angular extrusion under plane strain conditions. The metal is modelled as a Von-Mises material while the Mohr-Coulomb friction law is used for the metal-wall interaction. The analysis considers a 90° channel and three different interface conditions. The corresponding velocity fields are shown in Figure 3, together with the collapse pressures.



Figure 3. Equal-channel angular extrusion: FE mesh and velocity fields.

## REFERENCES

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