

Convergence Analysis of an Adaptive Interior Penalty Discontinuous Galerkin Method

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Key Words: *Interior Penalty Discontinuous Galerkin method, residual type a posteriori error estimators, convergence analysis.*

ABSTRACT

We are concerned with a convergence analysis of an adaptive symmetric Interior Penalty Discontinuous Galerkin (IPDG) method for second order elliptic boundary value problems. Based on a residual-type a posteriori error estimator, we prove that after each refinement step of the adaptive scheme we achieve a guaranteed reduction of the global discretization error in a mesh dependent energy norm associated with the IPDG method. In contrast to recent work on adaptive IPDG methods, the convergence analysis does not require multiple interior nodes for refined elements of the triangulation and thus leads to a more efficient adaptive scheme. In fact, it will be shown that bisection of elements is sufficient. The main ingredients of the proof of the error reduction property are the reliability and a perturbed discrete local efficiency, a bulk criterion that takes care of a proper selection of edges and elements for refinement, and a perturbed Galerkin orthogonality property with respect to the energy inner product. A documentation of numerical experiments is given to illustrate the performance of the adaptive method.