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SWELLING PHENOMENA IN ACTIVE SOIL

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ABSTRACT

Charged hydrated materials such as active soil exhibit internal coupling mechanisms stemming from the intrinsic characteristics of a soil matrix charged by negative ions in combination with a pore-fluid mixture consisting of a liquid solvent and negative as well as positive ions of a dissolved salt, cf. [1]. Generally, this kind of materials exhibiting a multicomponent microstructure can be descibed on the basis of the well-founded framework of the Theory of Porous Media (TPM), cf. [2–4]. The coupling of chemical, mechanical and electrical mechanisms in the overall aggregate initiates swelling mechanisms inducing finite deformations. Describing a swelling soil, the TPM model results in a binary aggregate of immiscible constituents, solid matrix (φ^S) and fluid phase (φ^F), extended by the incorporation of volume-free fixed charges (φ^{fc}) to the solid matrix. Furthermore, the fluid is treated as a mixture of miscible components, namely, a liquid solvent (φ^L), cations (φ^+) and anions (φ^-). Following this, and introducing the volume fractions n^α, which relate the differential volume element $\mathrm{d}v^\alpha$ of the constituents to the bulk volume element $\mathrm{d}v$ of the overall aggregate ($n^\alpha = \mathrm{d}v^\alpha/\mathrm{d}v$), the saturation condition is given by

$$\sum_\alpha n^\alpha = n^S + n^F = 1\,, \qquad \text{where} \quad n^F = \sum_\beta n^\beta = n^L + n^+ + n^-\,. \tag{1}$$

Proceeding from quasi-static processes with materially incompressible constituents, where mass exchanges among the constituents such as chemical reactions and phase transitions are excluded, the balance relations responsible for the chemo-mechanical process and the *Poisson* equation governing the electrical mechanisms can be introduced. In particular, the volume balance for the fluid phase φ^F, the concentration balances of the mobile ions φ^γ, the overall momentum balance and the *Poisson* equation read:

$$\begin{aligned} &\operatorname{div}[(\mathbf{u}_S)'_S + n^F \mathbf{w}_F] = 0\,, \qquad n^F\,(c^\gamma_m)'_S + c^\gamma_m \operatorname{div}(\mathbf{u}_S)'_S + \operatorname{div}(n^F c^\gamma_m \mathbf{w}_\gamma) = 0\,, \\ &\mathbf{0} = \operatorname{div}(\,\mathbf{T}^S_{E\,\text{mech.}} - p\,\mathbf{I}\,) + \rho\,\mathbf{b}\,, \qquad \operatorname{div}\operatorname{grad}\xi = \frac{n^F F}{\epsilon}\Big(\sum_\gamma z^\gamma c^\gamma_m + z^{fc} c^{fc}_m\Big)\,. \end{aligned} \tag{2}$$

Therein, $(\,\cdot\,)'_\alpha$ represents the material time derivative with respect to φ^α. Furthermore, $\mathbf{u}_S$ denotes the solid displacement, $\mathbf{w}_\alpha$ is the seepage velocity of φ^α, c^γ_m and z^γ are the molar concentration and the electrical valence of φ^γ, $\mathbf{T}^S_{E\,\text{mech.}}$ is the purely mechanical solid extra stress given by an extended neo-*Hooke*an model, and $\rho\,\mathbf{b}$ is the volume force acting on the whole aggregate. Moreover, ξ denotes the electrical potential, F the *Faraday* constant and ϵ the electrical permittivity. Note that p is the overall pore pressure consisting of two parts, the *Lagrange*an multiplier $\mathcal{P}$ as the purely hydraulic pressure given by the boundary conditions and the osmotic pressure π developing from concentration differences:

$$p := \mathcal{P} + \pi\,. \tag{3}$$

Going through the process of the evaluation of the entropy inequality, one obtains restrictions and conditions for the formulation of constitutive equations such as the equations for stress tensors and momentum productions (interaction forces between the constituents). Moreover, to capture the high non-linearities of ionic solutions, the concentration-dependent activity coefficients of the ions are considered. Based on the constitutive equations, a *Darcy*-type equation for the seepage velocity of the fluid and a *Nernst-Planck*-type equation for the ion velocities are derived. To simulate the behaviour of active soil under mechanical, chemical and electrical loads, the model is implemented into the FE tool PANDAS[1], where, to set an example, the swelling of an elliptic hydrogel disc is simulated. In order to obtain numerically stable solutions and to fulfill the LBB condition, use is made of mixed hexahedral *Taylor-Hood* elements, where the solid displacement $\mathbf{u}_S$ is approximated by quadratic shape functions, while the overall pressure p, the concentrations c^γ_m and the electrical potential ξ are approximated by linear shape functions. Note that this choice of primary variables initiates boundary conditions depending on internal variables. For stability reasons, this leads to the *Dirichlet* boundary conditions for p, c^γ_m as well as ξ to be imposed weakly.

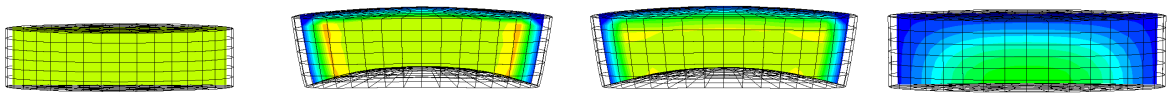

Figure 1: Numerical simulation by the FE tool PANDAS.

To initiate the swelling process, the concentration of the external solution is decreased at the upper and circumferential surface from 0.20 mol/l to 0.15 mol/l within 10 s. The contour plot in Figure 1 exhibits the decreasing cation concentration. As one can see, the specimen initially begins to swell in the upper and the circumferential boundary, thus yielding a bending of the specimen. At the end of the simulation, when equilibrium is reached and the volume of the overall aggregate is increased, the bending behaviour vanishes again.

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[1] **P**orous media **A**daptive **N**onlinear finite element solver based on **D**ifferential **A**lgebraic **S**ystems (`www.get-pandas.com`)