8th. World Congress on Computational Mechanics (WCCM8)
5th. European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 – July 5, 2008
Venice, Italy

BODY SHAPE RECONSTRUCTION BY ITS SCATTERED PATTERNS

*** O. Kusyi[1,2], V. Voitovich[2]**

[1] Kassel University
3 Kurt-Wolters str., Kassel 34127, Germany
okusyy@uni-kassel.de
okusij@mail.ru

[2] Pidstryhuch Institute for Applied Problems of Mechanics and Mathematics, NASU
3"B" Naukova str., Lviv 79060, Ukraine
voi@iapmm.lviv.ua

Key Words: *Inverse Scattering Problems, Orthogonal Complement Function, Herglotz Wave Function.*

ABSTRACT

A method of body shape reconstruction using the approximating Herglotz functions is developed. The method is based on properties of far-field patterns scattered on a body at eigenfrequencies of its interior volume. If the analytical continuation of the eigenoscillation field outside the body has no singularities and decays at infinity, then any set of far-field patterns is noncomplete. In this case, there exists the function orthogonal to this set [1, 2]. It is a kernel of the Herglotz wave function coinciding with the eigenoscillation inside the body and on its boundary and hence vanishing on the boundary identically. Otherwise, when the continuation has singularities or does not decay at infinity then such a Herglotz function does not exist, but the eigenoscillation inside the body can be approximated in appropriate fuctional space by a sequence of Herglotz functions [3].

The method consists in minimizing the functional formulated in a finite-dimensional space as

$$L_M(F) = \frac{1}{\|F\|^2} \sum_{m=1}^{M} \frac{|(f_m, F)|^2}{\|f_m\|^2}, \tag{1}$$

where f_m, $m = 1..M$ is a set of measured scattering patterns, F is the sought orthogonal complement function. The Herglotz function with the kernel F_M minimizing the functional (1) is then defined as

$$u_M(\mathbf{x}) = \int_{\Omega} e^{ik\mathbf{x}\cdot\mathbf{d}} F_M(\mathbf{d}) ds(\mathbf{d}), \tag{2}$$

where $\mathbf{d}$ is the point on infinitely remote sphere Ω, and $\mathbf{x}$ is any point in the free space.

Numerical results suggest to make an assertion that the Herglotz functions (2) possess the approximative property in the sense that the body boundary is approached by zero surfaces of u_M, as the number M of measured scattering patterns increases.

The proposed method was tested on a lot of two-dimensional problems [4, 5]. Fig. 1. shows reconstruction results for the body (dashed lines) with contour beeing a part of the zero-line net of the function $N_5(k\tilde{r})\cos(5\tilde{\varphi})$ (N_n is the Neumann function) having singularity outside the body (at $\tilde{r}=0$). The origin of polar coordinates $\tilde{r},\tilde{\varphi}$ is marked by asterisk in Fig 1a. It is seen that for such a body there exists no Herglotz function vanishing on the body boundary. Nevertheless, the zero lines of approximative

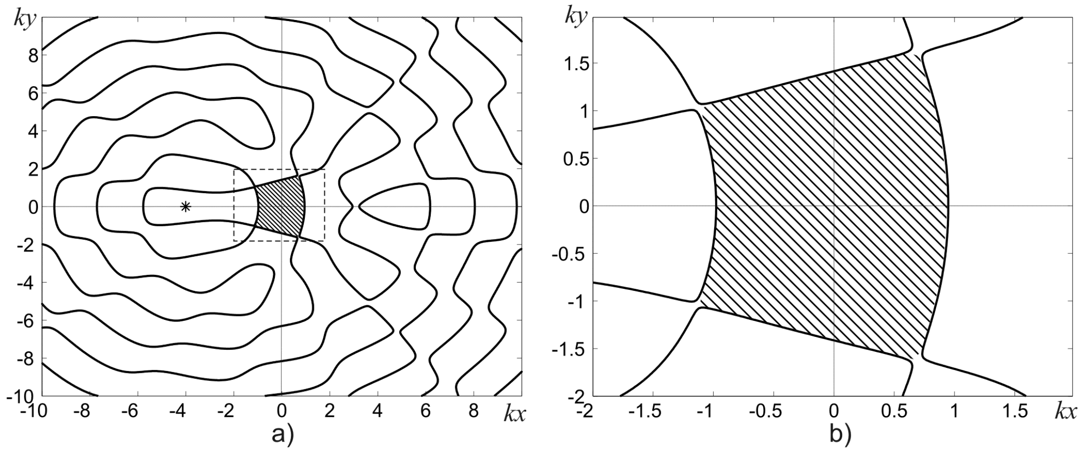

Fig. 1. Zero lines of approximating Herglotz function for body with singularity in exterior continuation of eigenoscillation field.

Herglotz function created by the proposed method reconstruct the body contour satisfactorily and differ from it only at the corners.

The following features of the proposed method make it competetive with existing methods of solving inverse scattering problems:
a) no need of solving direct problems;
b) considerably small amount of input data is required for satisfactory reconstruction of body contour (usually up to 10 scattering patterns in two-dimensional problems);
c) scattering patterns should not be a result of incidence of plane waves. They must only be linearly independent.

Modification of this method can also be applied for reconstuction of uniformly moving [6] or rotating bodies.

REFERENCES

[1] D. Colton, R. Kress, *Inverse Acoustic and Electromagnetic Scattering Theory*, Springer–Verlag, 1998.

[2] B. Z. Katsenelenbaum, *Electromagnetic Fields – Restrictions and Approximation*, WILEY– VCH, 2003.

[3] D. Colton, R. Kress, “On the denseness of Herglotz wave functions and electromagnetic Herglotz pairs in Sobolev spaces“, *Mathematical Methods in the Applied Sciences*, vol. **24**, pp. 1289–1303, (2001)

[4] O. Kusyi, N. Voitovich, “Body shape reconstruction by its scattering patterns. Influence of data distortion“, *Proc of DIPED Conf.*, pp. 189–192, (2006).

[5] O. Kusyi, N. Voitovich, “Properties of orthogonal complement functions in body shape reconstruction problems”, *Proc of DIPED Conf.*, pp. 122–125, (2007)

[6] O. Kusyi, N. Voitovich, “Shape reconstruction of uniformly moving body by its scattering patterns”, *36th European Microwave Conference*, p. 55, (2006).