**8th. World Congress on Computational Mechanics (WCCM8)**
**5th European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)**
**June 30 –July 5, 2008**
**Venice, Italy**

# An a posteriori error estimator for a hierarchical model dimension reduction

**Alexandre Ern**[1], *** Simona Perotto**[2] **and Alessandro Veneziani**[2]

[1] CERMICS, ENPC
Champs sur Marne
6 et 8, av. Blaise Pascal
Marne la Vallée Cedex 2
F-77455 France
ern@cermics.enpc.fr

[2] MOX
Dipartimento di Matematica
Politecnico di Milano
Via Bonardi 9
I-20133 Milano, Italy
simona.perotto@polimi.it
alessandro.veneziani@polimi.it

**Key Words:** *Model Reduction, Domain Decomposition, A Posteriori Modeling Error Estimator, Goal-Oriented Analysis.*

## ABSTRACT

This work is inspired by physical phenomena of different nature characterized by a dominant spatial direction. As examples we quote river dynamics and blood flows. In these problems, relevance of transversal (i.e., not aligned with the dominant direction) dynamics is basically related to local features of the problem, such as an obstacle or a lake in the river stream as well as an occlusion or an aneurysm in a vessel.

To model these situations we adopt a strategy alternative to the geometrical multiscale method ([3,4]). In the latter approach models with a different number of space variables are coupled. In our approach we couple 1D models with a different level of transversal detail. We simplify the reference problem (called *full model*) by tackling in a different manner the dependence of the solution on the leading direction and on the transverse ones. The former is spanned by a classical piecewise polynomial basis. The latter are expanded into a *modal basis*, which can be adapted according to the complexity of the transversal dynamics. We end up with a *hierarchy* of problems (called *reduced models*), distinguishing one another for the different number of transversal modal functions. In principle the reduced model can be tuned by a suitable selection of the modal basis, thus yielding an approximate solution to the full problem as accurate as desired. We refer to this new approach as *hierarchical model reduction* ([1,2]).

Final goal of this analysis is to devise a model-adaptive procedure, to automatically detect the number of modal functions to be included in the different parts of the domain. For this purpose we first set up an a posteriori modeling error analysis, yielding a hierarchical modeling error estimator. Then an efficient adaptive algorithm is proposed. A domain decomposition approach is used to enforce suitable matching conditions between the regions characterized by different modal bases.

## REFERENCES


[1] A. Ern, S. Perotto and A. Veneziani. "Hierarchical model reduction for advection-diffusion-reaction problems". Proceedings of ENUMATH 2007, the 6th European Conference on Numerical Mathematics and Advanced Applications, Graz 10–14 September 2007, submitted.

[2] A. Ern, S. Perotto and A. Veneziani. "Hierarchical model reduction for elliptic problems: a domain decomposition approach". In preparation.

[3] L. Formaggia, F. Nobile, A. Quarteroni and A. Veneziani. "Multiscale modelling of the circulatory system: a preliminary analysis". *Comput. Visual. Sci.*, Vol. **2**, 75–83, 1999.

[4] E. Miglio, S. Perotto and F. Saleri. "Model coupling techniques for free-surface flow problems. Part I". *Nonlinear Analysis*, Vol. **63**, 1885–1896, 2005.