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NUMERICAL SOLUTION OF 3D FRICTION CONTACT PROBLEM

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ABSTRACT

In this work we develop the variational method for solving of the contact problem, taking into account the dependence of the contact, stick and slip domain on the external load.
We consider the quasi-static contact problem for Coulomb's friction law. The relative velocities in Coulomb's law are chosen as derivatives of the displacements with respect to the variable t which is a length of a loading curve in a space of loading parameters.

The transformation of the local friction contact problem into a variational inequality is given. An iterative solution method is proposed together with the step-by-step procedure on the loading parameter, and with the spatial discretization by the boundary element method.

Exhaustive description and foundation of the mathematical formulation of the proposed formulation of friction contact problem is given in [1]. Proposed method for solution consists of the combination of step-by-step method with respect to a loading parameter and Uzawa's iterations developed for the solution of a saddle point search with inequality constraints. Solution for each loading step results in the solution of variational inequality:

$$a(u,v-u)-L(v-u)-\int_{\Sigma_C} f\mid\sigma_N(u)\mid(\mid v-u_T^t\mid-\mid u_T-u_T^t)d\Sigma\geq 0, \qquad (1)$$

where $\forall v\in K$, $u\in K$, u is the unknown displacement field for the loading parameter $t+dt$, u^t is the displacement field known from precedent loading step t, low index "T" denote the projection on the plane tangent to the boundary Σ of the deformed body Ω, σ_N is the contact pressure on the contact boundary Σ_C; f is the friction coefficient. The field v is an arbitrary cinematically admissible displacement satisfying the impenetrability condition; $a(u,v-u)$ is a bilinear form associated to the deformation work on the displacement

variations, $L(u-v)$ is the external forces work on same displacement variations. Dependence of the pressure σ_N on an unknown displacement u put obstacle in the transformation of the inequality (1) to a minimization problem. This problem is overcoming by iteration method in which the value $\sigma_N(u)$ is replaced with $\sigma_N(u^{(i)})$, where i is the iteration number, $u^{(0)}$ is assigned. Additionally, the following transformation is used: $f \mid \sigma_N(u^{(i)}) \mid \mid v_T \mid = \max\limits_{|\sigma_T| \le f|\sigma_N^{(i)}|} \sigma_T \cdot v_T$. Finally, we arrive to a sequence of the saddle point problems solved with Uzawa's algorithm.

Numerical solution of some 2D and 3D friction contact problems are obtained. Analysis of the numerical results gives important conclusions on the complex loading effects, cohesion domain evolution, friction contact tractions. For example, it was detected the reduction of the maximum friction traction due to presence of increments in (1). As examples the distribution of the pressure σ_N for a rigid cube is shown at the Fig.1, and the friction contact traction (modulus) for the finite cylinder indentation into an elastic half-space is shown at the Fig. 2.

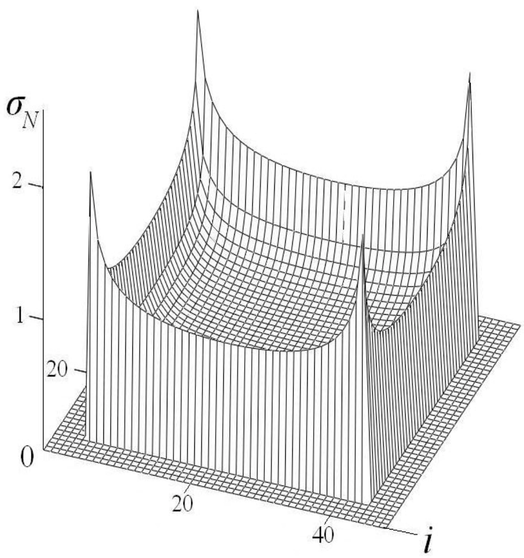

Fig. 1: Normal pressure σ_N for for the cube indenter

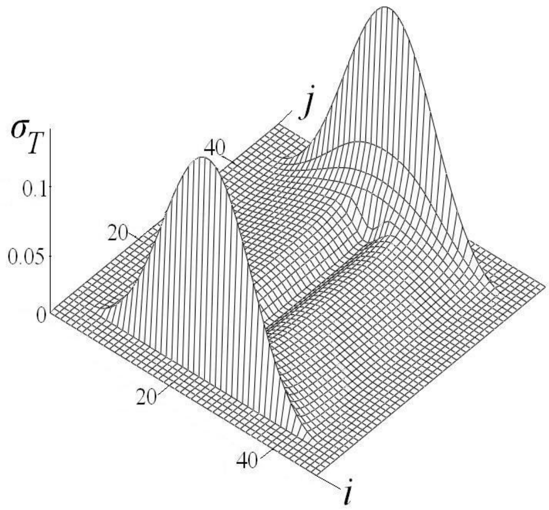

Fig. 2: Modulus of friction traction σ_T the finite cylinder indenter

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REFERENCES

[1] A.S.Kravchuk, P.Neittaanmäki *Variational and Quasi-Variational Inequalities in Mechanics*, Springer, 2007.